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DYNAMICAL SYSTEMS AND MICROPHYSICS: A WISH ;

DEDICATED TO THE COMPOSER AUREL STROE.

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DYNAMICAL SYSTEMS AND MICROPHYSICS: A WISH.

(Dedicated to the Composer Aurel Stroe)

S. Ciulli

Talk given at the Second International Seminar on Mathematical Theory of Dynamical Systems and Microphysics†.

(Extended Abstract)

1. INTRODUCTION

I am very grateful to Prof. A. Avez, Prof. A. Blaquière and to Prof. A. Marzollo for having invited me to give this talk. In fact, I would like to use this opportunity not for giving a lecture but merely to express a wish: I am indeed convinced that a good deal of the mathematical prerequisites necessary to tackle the questions discussed below are already known to the Dynamical Systems people, and I would be very happy if they would become active in this field.

Without any doubt the Greatest Miracle in Nature is that there are no Miracles at all (or only very few). Laws inferred from experiments performed for some range of values of some parameters are usually valid also in regions where no experiments were performed at all (in Future Time, for instance), and hence, surprisingly enough[1], these laws will usually have positive predictive power.

The zero - order (naive) way of answering this question is to do as some philosophers are doing, to state that "the World is cognoscible" and to stop there. I might have myself stopped my talk at this point if I had not had the enticing example of the XVI-th century mathematician, François Viete, who was able to find some simple rules (= elementary algebra) which were a ready-to-use sub-

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[1]*This "surprisingly enough" reflects the fact that we expect the metatheoretical world should have some mixing or ergodic features too. This note is here just to stress how suitable is the Dynamical System language for this kind of questions.*

stitute for all the endeavour and personal talent necessary to solve awkward classroom arithmetic problems. Will Dynamical Systems and related theories turn into the algebra of Natural Science Laws?

2. RUELLE-TAKENS CRITICS OF LANDAU'S THEORY OF TURBULENCE is a good example [1] of such a concern. In fact they have not dismissed Landau's theory, but have shown that his quasiperiodic-function approach is 'fragile' towards perturbations, and hence infinitely improbable. Indeed, since experiments are never able to determine individual points but rather work with diffuse open sets, physical concepts and laws (A) should be quite insensitive to small virtual perturbations, i.e. *structurally stable*.

A serious question arises at this point: how may one determine the class in which these virtual perturbations may lie? Since Natural Sciences are experimental Sciences we believe that this point has to be solved by experiment; in Section 5, while discussing Peixoto's symmetry breaking, we shall give a non-trivial example how that might be done.

3. CONTRADICTORY AIMS

Physical concepts have to cope not only with the imprecisions of the experiments but also with (B) the lack of knowledge of huge regions where no experiment has yet been done (e.g. at energies surpassing the present elementary particle accelerators). But a law which will be invariant to virtual changes both in regions A and B, will be an excellent description of the present knowledge, but will have no predictive value at all. A scientist would therefore like somehow in a contradictory way, to have laws stable versus the imprecisions of class (A), but extremely biased towards the region of interest (B).

Fortunately there exists some "levers" [2] by means of which one may control "the spread-off" of the predictions in (B). In this respect, Analytic Scattering Theory is an excellent playground, since analytic continuation off open curves has *both* the properties of being *unique* - if the data is absolutely exact - *and* to be *ill-posed*, i.e. giving extremely poor predictions if errors are present. In section 6 we shall show, in a sketchy way, how these 'predictivity levers' might be used for practical purposes.

4. BROKEN SYMMETRIES AND BRANCHING POINTS

There has been much interest in spontaneous broken symmetries for the last twenty years (for a modern approach, see L. Michel [3]. Together with any other sort of branching, they represent the critical points of the theories, where some supplementary information is needed in order to restore predictivity[2]. Sometimes these critical points look like the branching of the stream lines of a flow, and the supplementary information which restores determinism is of the form "go left" or "go right". In other cases, like in analytic continuation off open contours, the lines of flow inside the Banach Space are unique, but they diverge infinitely in the higher dimensions, which makes predictions impossible if errors are present (= "branchings of the second kind").

However the above examples do not exhaust the various forms of spontaneous branching: a non-trivial example will be discussed next.

5. PEIXOTO'S FLOW

A geodetic flow on a thorus is given by $d\theta/d\phi = c$, where c is a constant. Using a square chart ($0<\phi<2\pi$, $0<\theta<2\pi$) for the thorus (opposite points identified), a geodetic will be a straight line, disappearing, for instance at the right and appearing again at the corresponding point at the left, and so on. This line is closed

[2] *Louis Michel enumerates in [3] a few mechanisms by means of which bifurcations (symmetry breakings) creep into Physics: (ii) summation of infinite number of analytic terms (e.g. the 'thermodynamic limit'), (iii) renormalization in field theory, (iv) decomposition of G-invariant quantum states into pure states, and, (i) just plain bifurcations, due to the nonlinearity of the evolution laws (e.g. when some Reynolds-type parameter exceeds some critical value). We kindly refer the reader to the original papers [3].*

only if $c = p/q$ (where p and q are integers), but as this happens with zero probability, in general a geodetic will tint the square in an uniform gray, in perfect agreement with the symmetries of the thorus.

Until now, nothing unusual. But lets consider the generic (smooth) deformations of the thorus. The geodetics will now draw wavy lines, as their equation becomes $d\theta/d\phi = c + f(\phi,\theta)$. Again in complete agreement with the symmetries of the thorus *and* of the class of deformations, the geodetics will almost always not close but pass arbitrarely near to any point. But the unexpected and really strange result of Peixoto, is that this flow *has repulsors and attractors* whose equations are *almost always straight lines with* $d\theta^{a,r}/d\phi^{a,r} = p/q$ (!). In other words the symmetry of the problem will be spontaneously broken by the appearence of light - gray (the repulsors) and dark - gray (the attractors) strips !

This kind of symmetry breaking is entirely different (see René Thom, [4]) of any symmetry breaking mechanism used in Physics. It is interesting to notice that these light - gray and dark - gray patterns *will not* appear for *any* class of deformations of the initial equation (for instance they will not appear if $f(\phi,\theta)$ is a simple constant). Hence they act as a theoretical (infinite) magnifying glass, allowing us to infer experimentally of which class are the (unknown) perturbations which occur in Nature.

6. STABILITY IN ANALYTIC SCATTERING THEORY

Speaking about second class bifurcations (see Sect. 3) we said that there exist various kinds of informations which are redundant in the ideal zero - error case, but which have efficient stabilization properties for ill - posed problems with non - zero errors.

As an example let's take the continuation problem of the Scattering Amplitude off space - like data points (off a segment lying inside the holomorphy domain of the Amplitude) to time - like ones, i.e. to the cuts. This is obviously an ill - posed problem. However if

some special information[3] is available, in the form, say, of some function-bound for the derivative, the continuation problem is obviously stabilized.

But these stabilizing entities are useful even if the true value of the bound is unknown. I shall try to show that in a simple example [5].

Let $\delta_0[A]$ be the least value of the weighted L^2 - norm of the (tangent) derivative on the cuts of the Amplitude with respect to energy, still compatible with the data A and analyticity. The weight may be chosen as to emphasise the region where the search is performed. If there will be much physical "structure" in that region, δ_0 will be large. One could check now the relative likelihood of different hypothesises (new particles, i.e. second Riemann-sheet poles, etc.) by substracting the latter and see how much δ_0 has decreased if computed for these modified functions, A'. As computer experiments have shown, if the errors are not too large, there are many orders of magnitude between the δ_0 corresponding to the correct hypothesis and that of the false ones.

These techniques have been used already with much success for instance to find values for vital parameters in Quantum Chromodynamics [6] and, in many other problems where Analyticity plays a leading role.

Although this kind of applications is less unprejudiced than those where the bound is known, these entities turn out to be strong investigation devices. Anyhow, since δ_0 is a characteristic *of the entire* set of 'Amplitudes' compatible with given conditions, if this test fails no other criterion will work!

I wish to explain why I have decided to dedicate this paper to the great Rumanian Composer Aurel Stroe. I have always been impressed by the profoundness of his thoughts and his exploring spirit, but

[3] *This information is certainly redundant in the zero-error case when the continuation is unique and the bound is identically satisfied.*

now I have especially in mind his Canto Interrotto from his Concert for Harpsichord and Orchestra. The rules of this composition were chosen so as to restrict the latter — in absence of new themes coming from the exterior — to an Attractor of four notes repeated at infinity. Aurel Stroe tried to suggest in this way that an Ideal, an Artistic Idea, or a whole National Culture which is isolated from exterior inputs is foredoomed to shrink to a sterile Limit Cycle where no new structures are generated.

This applies also to Science.

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DISCUSSIONS

R. THOM (Bures-sur-Yvette):

Q.: We have to admit that we understand very little of what happens in Nature: on the one hand predictability is related to elaborated systems (elaborated evolution laws), but the more intricate a system is, the less its stability. In choosing its laws Nature often seems to disregard Structural Stability. It is, for instance, strange that a jet (a Lie Algebra) may determine a whole physical Group, defying stability arguments.

-A: I certainly agree with you. However, paraphrasing Mallory, I would say that the very existence of this Fundamental Miracle is enough of a challenge to try to investigate it.

I would not dare to impose either Structural Stability or any other attribute *on Nature itself*, but I think that these concepts are useful in devising optimal strategies for investigation: I recall the Gedankenexperiment involving Peixoto's flow, which might be used to distinguish between the classes in which the virtual perturbations (the unknown parts of a physical law) may lie.

Concerning Lie Algebras and Groups, I think that the algebraic structure confers some "rigidity" against small perturbations. For instance, (α) *if the Lie Algebra is given*, that is, if the structure constants are *precisely known*, the group is determined up to its topology. But on the other hand, the fundamental group involves only integers, and integers are stable against (small) perturbations.

Further, (β) one may conceive of the structure constants varying, in such a way that, for instance, a compact group may become non compact. I really do not know of any physical example of this kind (physical groups thought to be compact, which turned out not to be so), but I suppose that if this kind of instability has not been commonly observed, it is because the physical object observed usually is *not* the Lie Algebra, but the Symmetry Group itself.

Anyhow I suppose that the intrinsic algebraic structure of a group confers on it some "immunity" against perturbations, and I think it would be worthwhile to study this subject a bit further.

V. LEPETIC (Karlovac).

Q.: It has often been emphasized that true theories also have a pronounced aesthetic appeal, that "truth is beautiful". May you comment on that?

-A: I guess that this question transcends a little the scope of this meeting. I will nevertheless try to give an anthropomorphic explanation, which, of course, may or may not be true.

Aesthetic feelings might be a kind of "pattern recognition" which might be, for instance, implanted genetically in our minds. In the same way as the acknowledgement of the beauty of a girl might be just an instinctive recognition that she is genetically perfect.

I apologise for this crude schematization. Biology, however, provides a lot of examples where orientation behaviour of insects and birds, say, is genetically transmitted. It is therefore natural to think that some ability to sense orientation in the more abstract realm of concepts (e.g. in order to get an economical description of the environment and so on) may also be genetically transmitted, and hence, the aesthetic feelings might reflect this a priori weighting of concepts.

Of course, this is just a guess. All that is a bit in the sense of the aesthetics of Herder, where beauty appears as a harmony between the internal and the external worlds. Plato has been perhaps the first to assert that concepts may be "remembered" from a time before being born.

R. THOM (Bures sur Yvette)

Q.: I have not yet understood well the precise qualities the stabilizing entities should have.

-A: They transform ball - topologies into lens - topologies. As they effectively control the predictions range of theories, they are in the same time sensitive diagnosis devices. Caprini's paper on determining quantum chromodynamic parameters by means of assymptotic expansions which we have just discussed here is a good example in this respect.