

# A POINCARÉ GAUGE THEORY OF GRAVITATION IN MINKOWSKI SPACETIME

C. WIESENDANGER

*School of Theoretical Physics, Dublin Institute for Advanced Studies*

*10 Burlington Road, Dublin 4, Ireland*

E-mail: wie@stp.dias.ie

## ABSTRACT

The conventional role of spacetime geometry in the description of gravity is pointed out. Global Poincaré symmetry as an inner symmetry of field theories defined on a fixed Minkowski spacetime is discussed. Its extension to local  $\mathbf{P}$  gauge symmetry and the corresponding  $\mathbf{P}$  gauge fields are introduced. Their minimal coupling to matter is obtained. The scaling behaviour of the partition function of a spinor in  $\mathbf{P}$  gauge field backgrounds is computed. The corresponding renormalization constraint is used to determine a minimal gauge field dynamics.

## 1. Introduction

The great success of modern particle physics in the description of the microscopic interactions of elementary particles relies much on the concept of gauge field theories. Only within this framework it was possible to formulate consistent renormalizable quantum field theories for interacting fields in four dimensions [1]. This provided a strong motivation to review the gravitational interaction and its beautiful description in the general theory of relativity both at the classical and the quantum level from the point of view of gauge field theories [2].

One central task thereby is to identify the correct gauge group for gravity. To settle this question rather different propositions were made in the literature on gauge approaches to gravity and yielded much new insight in the structure of gravitational interactions [2]. The earliest attempt was centered about the Lorentz group  $\mathbf{L}$  [3] and was enlarged to an analysis of the full Poincaré group  $\mathbf{P}$  only a few years later [4]. Since then there were many other contributions based on  $\mathbf{P}$  [5]-[18], the translation group  $\mathbf{T}$  [19]-[22] or even larger groups, e.g. [23]-[36].

Another important task is to analyze the status of the spacetime manifold in the description of gravity [37], [38]. On one hand gauge theories in elementary particle physics may be quantized without spoiling renormalizability only on Minkowski spacetime. As soon as a non-trivial background geometry is introduced the usual perturbative approaches face serious difficulties [39]. On the other hand the description of gravity in the general theory of relativity is just given in geometrical terms affecting the structure of spacetime itself. Accordingly one may ask whether some new light is shed on the problems in quantizing gravity using a complementary description which

disentangles the structure of spacetime from gravitational physics.

To obtain such a description it is important to notice the purely conventional role of spacetime geometry in the description of the behaviour of matter pointed out by Poincaré already [40]. In fact two equivalent points of view are possible [41].

Either, one defines the line element  $ds^2$  to be of Minkowskian form. Accordingly, in a gravitational field material rods will shrink and clocks slow down w.r.t. this metric. Hence, one defines the geometry of spacetime to be Minkowskian whereas the behaviour of physical rods and clocks has to be determined by experiments.

Or, one defines rods or clocks to have one and the same length or period at any point of spacetime. Accordingly, a measurement of the line element  $ds^2$  using these rods and clocks will yield that spacetime is curved in general. This is the convention Einstein introduced to describe gravitation.

The general theory of relativity and its extensions are based on the second point of view [42] which is very natural as long as one is interested in the macroscopic aspects of gravitation [43]. Its limitation shows up at the quantum level. To extend a picture so intimately related to classical concepts such as rods and clocks to a simple microscopic understanding of gravitation is very difficult. In microphysics spacetime geometry enters only as a background concept necessary in defining a field theory. It cannot be subject to direct measurements in this context.

Hence, at the quantum level one is naturally led to the first point of view avoiding the problematic interrelation of spacetime structure and gravitational phenomena. Here free matter is described by local, causal fields defined on Minkowski spacetime and its interactions are introduced using the gauge principle which allows a far-reaching generalization of the connection between conservation laws and global symmetry requirements [1].

To obtain a gauge theory of gravitation we ensure the conservation of energy-momentum and angular momentum by the requirement of global covariance of the free matter field theory under the Poincaré group. We give a complementary formulation of  $P$  symmetry and its consequences in the form of an inner symmetry (section 2) suggesting an analogy to the description of the action of inner symmetry groups as groups of generalized 'rotations' in field space [1]. In particular the coordinate system used to specify the spacetime events is not affected anymore by  $P$  transformations.

We next introduce local  $P$  gauge transformations and demand the invariance of physical processes under those (section 3). This necessarily leads to the existence of gauge fields with definite behaviour under local  $P$  gauge transformations. Their coupling to any other field is essentially fixed as in the case of other gauge field theories (section 4).

To restrict the classical gauge field dynamics we demand consistency with renormalization properties of matter fields. In a renormalizable theory the anomalous contribution to a change of a matter partition function under rescaling may be absorbed in the classical actions for the different fields (e.g. [44]). As an example we

determine the change of the one-loop partition function for a Dirac spinor under rescaling (sections 5 and 6) and may accordingly fix a minimal gauge field action (section 6).

We work on Minkowski spacetime  $(\mathbf{R}^4, \eta)$  with Cartesian coordinates throughout, such that  $\eta = \text{diag}(1, -1, -1, -1)$ . Indices  $\alpha, \beta, \gamma, \dots$  from the first half of the Greek alphabet denote quantities defined on  $(\mathbf{R}^4, \eta)$  which transform covariantly w.r.t. the Lorentz group. They are correspondingly raised and lowered with  $\eta$ .

## 2. Global Poincaré symmetry as an inner symmetry

Let us consider a set of fields  $\varphi_j(x)$  with  $j = 1, \dots, n$  which are defined on Minkowski spacetime  $(\mathbf{R}^4, \eta)$  and belong to some representations of the Poincaré group. Their dynamics shall be specified by the action

$$S_M = \int d^4x \mathcal{L}_M(\varphi_j, \partial_\alpha \varphi_j) \quad (1)$$

where the Lagrangian  $\mathcal{L}_M$  is assumed to be real.

The usual conception of global Poincaré transformations, acting partly on spacetime and partly in inner field space, is expressed in the transformation formulae

$$\begin{aligned} x^\alpha &\longrightarrow x'^\alpha = x^\alpha + \varepsilon^\alpha + \omega^\alpha{}_\beta x^\beta, \\ \varphi_j(x) &\longrightarrow \varphi'_j(x') = \varphi_j(x) - \frac{i}{4} \omega^{\alpha\beta} \Sigma_{\alpha\beta} \varphi_j(x). \end{aligned} \quad (2)$$

$\delta x^\alpha = \varepsilon^\alpha + \omega^\alpha{}_\beta x^\beta$  is the change of  $x$  under the combination of a global infinitesimal spacetime translation and an infinitesimal Lorentz rotation,  $\delta \varphi_j = -\frac{i}{4} \omega^{\alpha\beta} \Sigma_{\alpha\beta} \varphi_j(x)$  the corresponding change of  $\varphi_j$  in field space.  $\Sigma_{\gamma\delta}$  are the representations of the generators of the Lie algebra  $\text{so}(1,3)$  in inner field space normalized to fulfil the commutation relations

$$[\Sigma_{\gamma\delta}, \Sigma_{\epsilon\zeta}] = 2i\{\eta_{\delta\epsilon} \Sigma_{\gamma\zeta} - \eta_{\delta\zeta} \Sigma_{\gamma\epsilon} + \eta_{\gamma\epsilon} \Sigma_{\zeta\delta} - \eta_{\gamma\zeta} \Sigma_{\epsilon\delta}\}. \quad (3)$$

If the action Eq. (1) does not change under the continuous transformations Eqs. (2) the field theory is globally Poincaré invariant. Accordingly, Noether's theorem yields a conserved vector quantity

$$J^\gamma = \Theta^\gamma{}_\alpha \cdot \varepsilon^\alpha + \frac{1}{2} \mathcal{M}^\gamma{}_{\alpha\beta} \cdot \omega^{\alpha\beta}. \quad (4)$$

As  $\varepsilon^\alpha$  and  $\omega^{\alpha\beta}$  vary independently the canonical energy momentum tensor

$$\Theta^\gamma{}_\alpha = \frac{\partial \mathcal{L}_M}{\partial(\partial_\gamma \varphi_j)} \cdot \partial_\alpha \varphi_j - \eta^\gamma{}_\alpha \cdot \mathcal{L}_M \quad (5)$$

and the canonical angular momentum tensor

$$\mathcal{M}^\gamma{}_{\alpha\beta} = \Theta^\gamma{}_{\alpha}x_\beta - \Theta^\gamma{}_{\beta}x_\alpha + \frac{i}{2} \frac{\partial \mathcal{L}_M}{\partial(\partial_\gamma \varphi_j)} \Sigma_{\alpha\beta} \varphi_j \quad (6)$$

are individually conserved.

As a complementary conception we now introduce global infinitesimal  $\mathbf{P}$  gauge transformations

$$\begin{aligned} x^\alpha &\longrightarrow x'^\alpha = x^\alpha \\ \varphi_j(x) &\longrightarrow \varphi'_j(x) = ((1 + \Theta)\varphi_j)(x) \end{aligned} \quad (7)$$

where the infinitesimal hermitean gauge operators are given by

$$\Theta = -\{\varepsilon^\gamma + \omega^{\gamma\delta}x_\delta\}\partial_\gamma - \frac{i}{4}\omega^{\gamma\delta}\Sigma_{\gamma\delta}. \quad (8)$$

Much in analogy to non-abelian gauge field theory  $\mathbf{P}$  acts as a Lie group of generalized 'phase rotations'  $(1 + \Theta)$  in field space only and leaves the spacetime coordinates  $x$  unchanged. Note that one can also decompose  $\Theta$  w.r.t. the the  $\mathfrak{p}$  algebra generators  $p_\gamma = i\partial_\gamma$  and  $m_{\gamma\delta} = i(x_\gamma\partial_\delta - x_\delta\partial_\gamma) + \frac{1}{2}\Sigma_{\gamma\delta}$  which emphasizes the aforementioned useful analogy even more [45].

Eqs. (7) define again a symmetry transformation of globally Poincaré invariant actions as the corresponding Lagrangian just picks up a total divergence under a  $\mathbf{P}$  gauge transformation

$$\begin{aligned} \mathcal{L}_M(\varphi'_j(x), \partial_\alpha \varphi'_j(x)) &= \mathcal{L}_M(\varphi_j(x), \partial_\alpha \varphi_j(x)) \\ &\quad - \{\varepsilon^\gamma + \omega^\gamma{}_\beta x^\beta\} \cdot \partial_\gamma \mathcal{L}_M(\varphi_j(x), \partial_\alpha \varphi_j(x)) \end{aligned} \quad (9)$$

which does not contribute to the action integral.

Hence, we are led to a complementary conception of Poincaré symmetry as a purely inner symmetry. The corresponding Noether symmetry current is found to be the same  $J^\gamma$  as in Eq. (4). This shows that the two global conceptions are equivalent w.r.t. their physical consequences. On the other hand it is conceptually easy to generalize global  $\mathbf{P}$  gauge transformations to local ones and to build up the corresponding gauge theory in analogy to the non-abelian case because the structure of spacetime and the action of the gauge group on the fields remain strictly separated.

### 3. Local $\mathbf{P}$ gauge invariance and the covariant derivative $\tilde{\nabla}_\alpha$

Let us extend  $\mathbf{P}$  to a Lie group of local infinitesimal gauge transformations by allowing  $\varepsilon(x)$  and  $\omega(x)$  to vary with  $x$ . We thus consider from now on

$$\Theta(x) = -\{\varepsilon^\gamma(x) + \omega^{\gamma\delta}(x)x_\delta\}\partial_\gamma - \frac{i}{4}\omega^{\gamma\delta}(x)\Sigma_{\gamma\delta}. \quad (10)$$

Note that the algebra of the  $\Theta(x)$ 's does close again. There is a new element of non-commutativity in their algebra as, contrary to the usual case, the local parameters  $\varepsilon(x)$  and  $\omega(x)$  do not commute with the generators  $\partial_\gamma$  of the algebra. The emerging ordering problem is overcome by the convention that  $\Theta(x)$  in its above form only acts to the right. This convention is motivated by demanding equivalence of the algebra of the  $\Theta(x)$ 's to the diffeomorphism times  $\text{so}(1,3)$  algebra. The formulae (7) still define the representation of  $\mathbf{P}$  in the space of fields.

Next, to recast a given matter theory in a locally  $\mathbf{P}$  gauge invariant form we must introduce a covariant derivative  $\tilde{\nabla}_\alpha$  which is defined by the requirement

$$\tilde{\nabla}'_\alpha (1 + \Theta(x)) = (1 + \Theta(x)) \tilde{\nabla}_\alpha. \quad (11)$$

Here  $\tilde{\nabla}'_\alpha$  denotes the gauge transformed covariant derivative. Because  $\tilde{\nabla}_\alpha$  transforms as a Lorentz vector we have to supplement the generators  $\Sigma_{\gamma\delta}$  of  $\text{so}(1,3)$  in matter field space occurring in the decomposition of  $\Theta(x)$  with the corresponding generators  $\Sigma_{\gamma\delta}$  acting on vectors to obtain the appropriate product representation as we will automatically do wherever necessary from now on.

We find that the Lagrangian with covariant derivatives  $\tilde{\nabla}_\alpha$  replacing the usual ones behaves under local infinitesimal  $\mathbf{P}$  transformations as

$$\begin{aligned} \mathcal{L}_M(\varphi'_j(x), \tilde{\nabla}'_\alpha \varphi'_j(x)) &= \mathcal{L}_M(\varphi_j(x), \tilde{\nabla}_\alpha \varphi_j(x)) \\ &- \{\varepsilon^\gamma(x) + \omega^{\gamma\delta}(x)x_\delta\} \cdot \partial_\gamma \mathcal{L}_M(\varphi_j(x), \tilde{\nabla}_\alpha \varphi_j(x)). \end{aligned} \quad (12)$$

Note that this does not yet ensure the local  $\mathbf{P}$  invariance of the original action  $S_M = \int \mathcal{L}_M$  because the second term in Eq. (12) is no longer a pure divergence as it was in the case of global infinitesimal transformations.

Now, to fulfil Eq. (11) we use the ansatz

$$\partial_\alpha \longrightarrow \tilde{\nabla}_\alpha = e_\alpha{}^\gamma \partial_\gamma + \frac{i}{4} B_\alpha{}^{\gamma\delta} \Sigma_{\gamma\delta} \quad (13)$$

decomposing  $\tilde{\nabla}_\alpha$  w.r.t.  $\partial_\gamma$  and  $\Sigma_{\gamma\delta}$  in the same way as the local gauge operator  $\Theta(x)$  in Eq. (10). This ansatz is compatible with the required behaviour Eq. (11) of the covariant derivative under local  $\mathbf{P}$  gauge transformations provided the 16 compensating translation gauge fields  $e_\alpha{}^\gamma$  transform as

$$\begin{aligned} \delta e_\alpha{}^\gamma &\equiv e'_\alpha{}^\gamma - e_\alpha{}^\gamma = e_\alpha{}^\zeta \cdot \partial_\zeta \{\varepsilon^\gamma + \omega^{\gamma\delta} x_\delta\} \\ &- \{\varepsilon^\zeta + \omega^{\zeta\eta} x_\eta\} \cdot \partial_\zeta e_\alpha{}^\gamma + \omega_\alpha{}^\zeta e_\zeta{}^\gamma \end{aligned} \quad (14)$$

and the 24 Lorentz gauge fields  $B_\alpha{}^{\gamma\delta}$  as

$$\begin{aligned} \delta B_\alpha{}^{\gamma\delta} &\equiv B'_\alpha{}^{\gamma\delta} - B_\alpha{}^{\gamma\delta} = e_\alpha{}^\zeta \cdot \partial_\zeta \omega^{\gamma\delta} - \{\varepsilon^\zeta + \omega^{\zeta\eta} x_\eta\} \cdot \partial_\zeta B_\alpha{}^{\gamma\delta} \\ &+ \omega_\alpha{}^\zeta B_\zeta{}^{\gamma\delta} + \omega^\gamma{}_\zeta B_\alpha{}^{\zeta\delta} + \omega^\delta{}_\zeta B_\alpha{}^{\gamma\zeta}. \end{aligned} \quad (15)$$

As in our conception coordinate and P gauge transformations are strictly separated we emphasize that the introduction of  $e_\alpha^\gamma$  and  $B_\alpha^{\gamma\delta}$  has neither implications on the structure of the underlying spacetime which we assumed to be  $(\mathbb{R}^4, \eta)$  endowed with the Minkowski metric  $\eta$ . Nor has it implications on the maximal symmetry group of  $(\mathbb{R}^4, \eta)$ , which is the Poincaré group if we still restrict ourselves to the use of Cartesian coordinates only.

With the abbreviations

$$d_\alpha \equiv e_\alpha^\gamma \partial_\gamma, \quad B_\alpha \equiv \frac{i}{4} B_\alpha^{\gamma\delta} \Sigma_{\gamma\delta}, \quad (16)$$

where  $\Sigma_{\gamma\delta}$  must be properly adjusted to the Lorentz group representation it acts upon, we write

$$\tilde{\nabla}_\alpha = d_\alpha + B_\alpha \quad (17)$$

from now on.  $d_\alpha$  is just the translation covariant derivative introduced in [20]. We finally remark that  $\tilde{\nabla}_\alpha$  may alternatively be decomposed w.r.t. the p algebra generators  $p_\gamma$  and  $m_{\gamma\delta}$  yielding the most convenient starting point for perturbation expansions [45].

#### 4. The field strength operator. Minimal coupling to matter fields

Before turning to the field strength operator itself we introduce the non P covariant translation field strength

$$[d_\alpha, d_\beta] \equiv H_{\alpha\beta}^\gamma d_\gamma \quad (18)$$

as in [20].  $H_{\alpha\beta}^\gamma$  is expressed in terms of  $e_\alpha^\gamma$  as [20]

$$H_{\alpha\beta}^\gamma = e^{-1\gamma}_\epsilon (e_\alpha^\zeta \cdot \partial_\zeta e_\beta^\epsilon - e_\beta^\zeta \cdot \partial_\zeta e_\alpha^\epsilon) \quad (19)$$

where  $e^{-1\gamma}_\epsilon$  is the matrix inverse to  $e_\alpha^\epsilon$ , i.e.  $e_\alpha^\epsilon \cdot e^{-1\gamma}_\epsilon = \delta_\alpha^\gamma$ .

This allows us now to obtain the field strength operator and its decomposition in a simple way. Taking into account the vector character of  $\tilde{\nabla}_\alpha$  a little algebra yields

$$\begin{aligned} S_{\alpha\beta} \equiv [\tilde{\nabla}_\alpha, \tilde{\nabla}_\beta] &= H_{\alpha\beta}^\gamma d_\gamma - (B_{\alpha\beta}^\gamma - B_{\beta\alpha}^\gamma) d_\gamma \\ &+ d_\alpha B_\beta - d_\beta B_\alpha + [B_\alpha, B_\beta]. \end{aligned} \quad (20)$$

Introducing the tensor coefficients of  $d_\gamma$

$$T_{\alpha\beta}^\gamma \equiv B_{\alpha\beta}^\gamma - B_{\beta\alpha}^\gamma - H_{\alpha\beta}^\gamma \quad (21)$$

we may rewrite  $S_{\alpha\beta}$  as

$$[\tilde{\nabla}_\alpha, \tilde{\nabla}_\beta] = -T_{\alpha\beta}^\gamma \tilde{\nabla}_\gamma + \frac{i}{4} \tilde{R}^{\gamma\delta}_{\alpha\beta} \Sigma_{\gamma\delta}, \quad (22)$$

where  $\tilde{R}^{\gamma\delta}{}_{\alpha\beta}$  is found to be

$$\begin{aligned}\tilde{R}^{\gamma\delta}{}_{\alpha\beta} &\equiv d_\alpha B_\beta{}^{\gamma\delta} - d_\beta B_\alpha{}^{\gamma\delta} + B_\alpha{}^{\delta\epsilon} B_{\beta\epsilon}{}^\gamma \\ &\quad - B_\beta{}^{\delta\epsilon} B_{\alpha\epsilon}{}^\gamma - H_{\alpha\beta}{}^\epsilon B_\epsilon{}^{\gamma\delta}.\end{aligned}\quad (23)$$

As  $S_{\alpha\beta}$  has a decomposition w.r.t.  $\tilde{\nabla}_\delta$  and  $\Sigma_{\gamma\delta}$  it acts in general not only as a matrix but also as a first order differential operator in field space.

From Eq. (21) we see that only if  $B_\alpha{}^{\gamma\delta}$  is related to  $H_{\alpha\beta}{}^\gamma$  the tensor  $T_{\alpha\beta}{}^\gamma$  may vanish. Denoting this particular  $B_\alpha{}^{\gamma\delta}$  with  $C_\alpha{}^{\gamma\delta}$  the required relation becomes

$$C_{\alpha\beta}{}^\gamma - C_{\beta\alpha}{}^\gamma = H_{\alpha\beta}{}^\gamma.\quad (24)$$

We may now solve for  $C_\alpha{}^{\gamma\delta}$  in terms of  $H_{\alpha\beta}{}^\gamma$  with the result

$$C_\alpha{}^{\gamma\delta} = \frac{1}{2} \left( H_\alpha{}^{\gamma\delta} - H_\alpha{}^{\delta\gamma} - H^{\gamma\delta}{}_\alpha \right).\quad (25)$$

Whenever  $B_\alpha{}^{\gamma\delta} = C_\alpha{}^{\gamma\delta}$ , i.e.  $T_{\alpha\beta}{}^\gamma = 0$  we omit the tilde, hence writing

$$\nabla_\alpha \equiv d_\alpha + C_\alpha.\quad (26)$$

Obviously we obtain now for  $S_{\alpha\beta}$  a matrix only

$$[\nabla_\alpha, \nabla_\beta] = \frac{i}{4} \tilde{R}^{\gamma\delta}{}_{\alpha\beta} \Sigma_{\gamma\delta}.\quad (27)$$

By construction  $S_{\alpha\beta}$  transforms homogeneously under infinitesimal local P gauge transformations

$$S'_{\alpha\beta} (1 + \Theta(x)) = (1 + \Theta(x)) S_{\alpha\beta}\quad (28)$$

leading to

$$\delta T_{\alpha\beta}{}^\gamma = \Theta(x) T_{\alpha\beta}{}^\gamma, \quad \delta \tilde{R}^{\gamma\delta}{}_{\alpha\beta} = \Theta(x) \tilde{R}^{\gamma\delta}{}_{\alpha\beta}.\quad (29)$$

$T_{\alpha\beta}{}^\gamma$  and  $\tilde{R}^{\gamma\delta}{}_{\alpha\beta}$  transform homogeneously under infinitesimal local P gauge transformations. We emphasize that  $T_{\alpha\beta}{}^\gamma = 0$  is indeed a gauge covariant statement as we implicitly assumed above introducing  $C_\alpha{}^{\gamma\delta}$ .

It is very convenient to introduce the homogeneously transforming difference of the two gauge fields

$$K_\alpha{}^{\gamma\delta} \equiv B_\alpha{}^{\gamma\delta} - C_\alpha{}^{\gamma\delta}\quad (30)$$

which is related to  $T_{\alpha\beta}{}^\gamma$  as

$$K_{\alpha\beta}{}^\gamma - K_{\beta\alpha}{}^\gamma = T_{\alpha\beta}{}^\gamma\quad (31)$$

with the obvious inversion

$$K_\alpha{}^{\gamma\delta} = \frac{1}{2} \left( T_\alpha{}^{\gamma\delta} - T_\alpha{}^{\delta\gamma} - T^{\gamma\delta}{}_\alpha \right).\quad (32)$$

We can express now  $\tilde{R}^{\gamma\delta}{}_{\alpha\beta}$  in terms of  $R^{\gamma\delta}{}_{\alpha\beta}$ , which we take as our fundamental field strength variable, and  $K_{\alpha}{}^{\gamma\delta}$  as

$$\begin{aligned}\tilde{R}^{\gamma\delta}{}_{\alpha\beta} &= R^{\gamma\delta}{}_{\alpha\beta} + \nabla_{\alpha}K_{\beta}{}^{\gamma\delta} - \nabla_{\beta}K_{\alpha}{}^{\gamma\delta} \\ &+ K_{\alpha}{}^{\delta\epsilon}K_{\beta\epsilon}{}^{\gamma} - K_{\beta}{}^{\delta\epsilon}K_{\alpha\epsilon}{}^{\gamma}.\end{aligned}\quad (33)$$

Next, let us turn to the extension of globally  $\mathbf{P}$  invariant matter actions to locally gauge invariant ones. In the previous section we have obtained the covariant derivative  $\tilde{\nabla}_{\alpha}$  yielding the behaviour Eq. (12) of a matter field Lagrangian under local  $\mathbf{P}$  gauge transformations which is not yet sufficient for the original action  $S_M = \int \mathcal{L}_M$  to be locally  $\mathbf{P}$  gauge invariant. Completing the Lagrangian with  $\det e^{-1}$

$$\det e^{-1} \cdot \mathcal{L}_M(\varphi_j, \tilde{\nabla}_{\alpha}\varphi_j) \quad (34)$$

we find that the combination Eq. (34) changes under a local  $\mathbf{P}$  gauge transformation by a pure divergence only

$$\begin{aligned}\det e'^{-1} \cdot \mathcal{L}_M(\varphi'_j, \tilde{\nabla}'_{\alpha}\varphi'_j) &= \det e^{-1} \cdot \mathcal{L}_M(\varphi_j, \tilde{\nabla}_{\alpha}\varphi_j) \\ &- \partial_{\gamma} \left( \{ \varepsilon^{\gamma}(x) + \omega^{\gamma\delta}(x)x_{\delta} \} \det e^{-1} \cdot \mathcal{L}_M(\varphi_j, \tilde{\nabla}_{\alpha}\varphi_j) \right).\end{aligned}\quad (35)$$

Therefore the minimally extended locally  $\mathbf{P}$  gauge invariant matter action finally becomes

$$S_M = \int d^4x \det e^{-1}(x) \cdot \mathcal{L}_M(\varphi_j(x), \tilde{\nabla}_{\alpha}\varphi_j(x)). \quad (36)$$

Of course,  $S_M$  remains invariant if we change from one to another inertial system by global coordinate translations or Lorentz rotations.

It is the conception of  $\mathbf{P}$  symmetry as an inner symmetry together with the gauge principle which has led us to this minimal coupling prescription. In this conception the gauge fields and their transformation behaviour do not interfere with the spacetime structure  $(\mathbf{R}^4, \eta)$  fixed by an a priori convention and the underlying geometry remains separated from the physics described by the  $\mathbf{P}$  gauge fields in the same manner it remains separated from the physics described by any usual matrix gauge field.

We remark that a geometric re-interpretation of the gauge fields and their corresponding field strengths introduced above may be given in the framework of Riemannian geometry [45]. But then the gauge group  $\mathbf{P}$  and the requirement of local  $\mathbf{P}$  gauge invariance are replaced by the groups of general (infinitesimal) coordinate transformations and local  $SO(1,3)$  frame rotations and the requirement of invariance under these groups [42], [43] and the geometry of spacetime is necessarily linked with these complementary symmetry requirements.

## 5. Dirac partition function in gauge field backgrounds

In Yang-Mills gauge field theory one may fix the gauge field dynamics quite uniquely by demanding gauge invariance of the action and using dimensional ar-



guments related to the renormalization properties of the theory. In a similar fashion we attempt here to obtain information on the classical P gauge field dynamics studying quantized matter fields and their renormalization properties in gauge field backgrounds. In the Yang-Mills case our arguments lead straightforward to the usual Yang-Mills action.

The assumption that the interactions of the P gauge fields with the different matter fields are renormalizable imposes strong conditions on the classical gauge field dynamics. For let us suppose that a given theory for a matter field and the gauge fields  $e_\alpha{}^\gamma$  and  $B_\alpha{}^{\gamma\delta}$  is perturbatively renormalizable. Then we know that the change of the partition function of the whole system under rescaling can be absorbed in its classical action yielding at most a nontrivial scale dependence of the different couplings, masses and wavefunction normalizations [44]. Hence, the explicit computation of the change of the one-loop matter partition functions under rescaling will allow us to constrain the classical gauge field dynamics [20].

For brevity we restrict ourselves to the non-trivial case of a Dirac spinor field, the analogous discussions of a scalar and a vector field theory are found in [45]. The globally P invariant action for a Dirac spinor with real Lagrangian is given by

$$S_M = \int d^4x \left\{ \frac{i}{2} \bar{\psi} \gamma^\alpha (\partial_\alpha \psi) - \frac{i}{2} \overline{(\partial_\alpha \psi)} \gamma^\alpha \psi - m \bar{\psi} \psi \right\}. \quad (37)$$

The Dirac matrices fulfil the usual Clifford algebra  $\{\gamma_\alpha, \gamma_\beta\} = 2\eta_{\alpha\beta}$  and the  $\text{so}(1,3)$  generators become  $\Sigma_{\alpha\beta} = \frac{i}{2}[\gamma_\alpha, \gamma_\beta]$ . The minimal extension prescription yields the locally P gauge invariant action

$$S_M = \int d^4x \det e^{-1} \left\{ \frac{i}{2} \bar{\psi} \gamma^\alpha (\bar{\nabla}_\alpha \psi) - \frac{i}{2} \overline{(\bar{\nabla}_\alpha \psi)} \gamma^\alpha \psi - m \bar{\psi} \psi \right\}. \quad (38)$$

Due to spin  $B_\alpha{}^{\gamma\delta}$  enters the action. Partially integrating  $\bar{\nabla}_\alpha$  in the second term above leads to the usual form of the Dirac action

$$S_M = \int d^4x \det e^{-1} \bar{\psi} \left\{ i \gamma^\alpha (\bar{\nabla}_\alpha - \frac{1}{2} T_{\gamma\alpha}{}^\gamma) - m \right\} \psi. \quad (39)$$

Note the occurrence of the tensor  $T$  ensuring the hermiticity of the P covariant Dirac operator w.r.t.  $(\chi, \psi)_e = \int d^4x \det e^{-1} \bar{\chi} \cdot \psi$ .

The spinor partition function in the given gauge field background is given by the Grassmann functional integral

$$\mathcal{Z}_\psi[e, B] = \int \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{iS_M(\bar{\psi}, \psi; e, B)}. \quad (40)$$

Note that we omit possible normalizations in order to obtain the most general renormalization structure later. As  $S_M$  is already of the usual quadratic form we may perform the Grassmann integral and formally obtain

$$\mathcal{Z}_\psi[e, B] = e^{\frac{i}{2} \log \det M_\psi(e, B)}. \quad (41)$$

The hyperbolic fluctuation operator  $M_\psi(e, B)$  is obtained as usual by squaring the Dirac operator introduced in Eq. (39)

$$M_\psi(e, B) \equiv -\gamma^\alpha(\bar{\nabla}_\alpha - \frac{1}{2}T_{\gamma\alpha}{}^\gamma) \cdot \gamma^\beta(\bar{\nabla}_\beta - \frac{1}{2}T_{\delta\beta}{}^\delta) - m^2 \quad (42)$$

and is hermitean w.r.t.  $(,)_e$  due to the occurrence of  $T$ . To make contact to the heat kernel techniques fully described in [45] we have to recast  $M_\psi$ . Using  $[\bar{\nabla}_\alpha, \gamma^\beta] = 0$  and  $\gamma^\alpha\gamma^\beta = \eta^{\alpha\beta} - i\Sigma^{\alpha\beta}$  we obtain

$$\begin{aligned} M_\psi(e, B) &= -(\bar{\nabla}_\alpha - \frac{1}{2}T_{\gamma\alpha}{}^\gamma)(\bar{\nabla}^\alpha - \frac{1}{2}T_\delta{}^{\alpha\delta}) \\ &+ \frac{i}{2}\Sigma^{\alpha\beta}(-T_{\alpha\beta}{}^\delta\bar{\nabla}_\delta + \frac{i}{4}\bar{R}^{\gamma\delta}{}_{\alpha\beta}\Sigma_{\gamma\delta} - \bar{\nabla}_\alpha K_{\delta\beta}{}^\delta) - m^2. \end{aligned} \quad (43)$$

Next we write  $T_\alpha \equiv \frac{i}{4}T^{\gamma\delta}{}_\alpha\Sigma_{\gamma\delta}$  and absorb the first order derivative term  $-2T_\alpha\bar{\nabla}^\alpha$  in the second order one. Together with the use of the Jacobi identities for the covariant derivative  $\bar{\nabla}_\alpha$  we then find the manifestly hermitean result

$$\begin{aligned} M_\psi(e, B) &= -(\bar{\nabla}_\alpha + T_\alpha - \frac{1}{2}T_{\gamma\alpha}{}^\gamma)(\bar{\nabla}^\alpha + T^\alpha - \frac{1}{2}T_\delta{}^{\alpha\delta}) \\ &+ T_\alpha T^\alpha + \frac{i}{2}\Sigma^{\alpha\beta}(\frac{i}{4}\bar{R}^{\gamma\delta}{}_{\alpha\beta}\Sigma_{\gamma\delta} + \bar{R}^\delta{}_{\alpha\delta\beta}) - m^2. \end{aligned} \quad (44)$$

This is the form relevant for further computation.

As we are interested in the behaviour of  $\mathcal{Z}_\psi[e, B]$  under rescaling the most suited renormalization of the ultraviolet divergent determinant in Eq. (41) is based on the  $\zeta$ -function as it is a manifestly gauge invariant technique.

One can define the ultraviolet regularized functional determinant of an operator  $M$  satisfying certain conditions to be [46], [47]

$$\log \det M \equiv -\lim_{u \rightarrow 0} \frac{d}{du} \zeta(u; \mu; M) \quad (45)$$

where the generalized  $\zeta$ -function belonging to  $M$  is given by the Mellin transformed of the heat kernel

$$\zeta(u; \mu; M) = \frac{i\mu^{2u}}{\Gamma(u)} \int_0^\infty ds (is)^{u-1} \text{Tr} e^{-isM}. \quad (46)$$

The scale  $\mu$  at which parameters such as couplings, masses and wavefunction normalizations have to be adjusted is introduced in order to keep the determinant dimensionless.

Hence, with the use of Eq. (45) the spinor partition function normalized at scale  $\mu$  becomes

$$\mathcal{Z}_\psi[\mu; e, B] = e^{-\frac{1}{2}\zeta'(0; \mu; M_\psi(e, B))}. \quad (47)$$

According to the formula

$$\zeta'(0; \tilde{\mu}; M) = \zeta'(0; \mu; M) + 2 \log \lambda \cdot \zeta(0; \mu; M). \quad (48)$$

we finally obtain the change of  $\mathcal{Z}_\psi$  corresponding to a change of scale  $\tilde{\mu} = \lambda\mu$

$$\mathcal{Z}_\psi[\tilde{\mu}; e, B] = \mathcal{Z}_\psi[\mu; e, B] \cdot e^{-\log \lambda \cdot \zeta(0; \mu; M_\psi(e, B))}. \quad (49)$$

## 6. Scaling of the Dirac partition function and the dynamics of the gauge fields

In the previous section we expressed the change of the one-loop spinor partition function under rescaling in terms of the  $\zeta$ -function belonging to the corresponding fluctuation operator. Renormalizability of any theory including dynamical gauge fields requires now at least that this anomalous contribution, which is a local polynomial in  $e_\alpha{}^\gamma$  and  $B_\alpha{}^{\gamma\delta}$  and their derivatives, may be absorbed in the classical action for the gauge fields  $e_\alpha{}^\gamma$  and  $B_\alpha{}^{\gamma\delta}$  [44]. Hence, to determine a minimal gauge field dynamics consistent with renormalizability we finally have to obtain explicitly the value of the  $\zeta$ -function at zero.

In the representation Eq. (46) for  $\zeta(u; \mu; M)$  it is the singular part of the  $s$ -integration which yields a nonvanishing value for  $\zeta(0; \mu; M)$ . As this singular part comes from the small  $s$ -region we may use the corresponding expansion for the trace of the heat kernel [44]

$$\text{Tr} e^{-isM} \underset{s \rightarrow 0}{\sim} \frac{i}{(4\pi is)^{\frac{d}{2}}} \sum_{k=0}^{\infty} (is)^k \int d^d x \det e^{-1} \text{tr} c_k(x) \quad (50)$$

given in terms of the well-known Seeley–DeWitt coefficient functions  $c_k$ . Performing the  $s$ -integration in (46) singles out the contribution for  $k = \frac{d}{2}$  from the infinite sum and one obtains [44]

$$\zeta(0; \mu; M) = \frac{i}{(4\pi)^{\frac{d}{2}}} \int d^d x \det e^{-1} \text{tr} c_{\frac{d}{2}}(x). \quad (51)$$

The computation of the  $c_{\frac{d}{2}}$  has been done in various ways in the literature [49]–[53] and we restrict ourselves to give the result relevant to our case, where  $d = 4$  and  $\text{tr}_D$  denotes the Dirac trace [45]

$$\begin{aligned} \text{tr}_D c_2 &= 4U_m + \left(\frac{1}{6} R^{\alpha\beta}{}_{\alpha\beta} - m^2\right) \cdot (4V_3 - V_1^{\gamma\delta}{}_{\gamma\delta}) \\ &- \frac{1}{6} \nabla_\alpha \nabla^\alpha (4V_3 - V_1^{\gamma\delta}{}_{\gamma\delta}) - \frac{1}{24} F_{\alpha\beta\gamma\delta} \cdot F^{\alpha\beta\gamma\delta} \\ &- V_3 \cdot V_1^{\gamma\delta}{}_{\gamma\delta} - \frac{1}{8} V_{2\alpha\beta} \cdot V_2^{\alpha\beta} + \frac{1}{8} V_1^{\alpha\beta}{}_{\alpha\beta} \cdot V_1^{\gamma\delta}{}_{\gamma\delta} \\ &+ 2V_3^2 + \frac{1}{8} V_{1\alpha\beta\gamma\delta} \cdot (V_1^{\alpha\beta\gamma\delta} + V_1^{\gamma\delta\alpha\beta} + V_1^{\beta\delta\alpha\gamma}). \end{aligned} \quad (52)$$

The term without  $T$ -dependence

$$\begin{aligned}
U_m &= -\frac{1}{30}\nabla_\gamma\nabla^\gamma R^{\alpha\beta}{}_{\alpha\beta} + \frac{1}{72}R_{\alpha\beta}{}^{\alpha\beta} \cdot R_{\gamma\delta}{}^{\gamma\delta} \\
&+ \frac{1}{180}R_{\alpha\beta\gamma\delta} \cdot R^{\alpha\beta\gamma\delta} - \frac{1}{180}R_{\alpha\gamma}{}^\alpha{}_\delta \cdot R_\beta{}^{\gamma\beta\delta} \\
&- \frac{1}{6}m^2 \cdot R^{\alpha\beta}{}_{\alpha\beta} + \frac{1}{2}m^4
\end{aligned} \tag{53}$$

already occurs in the case of a scalar field, whereas the terms containing  $T$

$$\begin{aligned}
V_{1\gamma\delta\alpha\beta} &= \tilde{R}_{\gamma\delta\alpha\beta} + \frac{1}{2}T_{\gamma\delta\eta}T_{\alpha\beta}{}^\eta, \\
V_{2\alpha\beta} &= \frac{1}{2}(V_1{}^\eta{}_{\alpha\eta\beta} - V_1{}^\eta{}_{\beta\eta\alpha}), \\
V_3 &= \frac{1}{2}\tilde{\nabla}_\alpha T_\gamma{}^{\alpha\gamma} - \frac{1}{4}T_{\gamma\alpha}{}^\gamma T_\delta{}^{\alpha\delta}
\end{aligned} \tag{54}$$

and

$$\begin{aligned}
F_{\gamma\delta\alpha\beta} &= \tilde{R}_{\gamma\delta\alpha\beta} + \tilde{\nabla}_\alpha T_{\gamma\delta\beta} - \tilde{\nabla}_\beta T_{\gamma\delta\alpha} \\
&+ T_{\alpha\beta}{}^\eta T_{\gamma\delta\eta} + T^\eta{}_{\gamma\alpha} T_{\eta\delta\beta} - T^\eta{}_{\gamma\beta} T_{\eta\delta\alpha}
\end{aligned} \tag{55}$$

are due to the spin of the Dirac field. As the result Eq. (52) expressed in terms of the natural variables  $R, T$  becomes algebraically tedious we give it in these variables only for the case  $T = 0$  [48]

$$\begin{aligned}
\text{tr}_{DC_2} &= \frac{1}{30}\nabla_\gamma\nabla^\gamma R^{\alpha\beta}{}_{\alpha\beta} + \frac{1}{72}R_{\alpha\beta}{}^{\alpha\beta} \cdot R_{\gamma\delta}{}^{\gamma\delta} \\
&- \frac{7}{360}R_{\alpha\beta\gamma\delta} \cdot R^{\alpha\beta\gamma\delta} - \frac{1}{45}R_{\alpha\gamma}{}^\alpha{}_\delta \cdot R_\beta{}^{\gamma\beta\delta} \\
&+ \frac{1}{3}m^2 \cdot R^{\alpha\beta}{}_{\alpha\beta} + 2m^4.
\end{aligned} \tag{56}$$

Insertion of the results Eq. (52) or Eq. (56) into Eq. (51) with  $d = 4$  finally yields  $\zeta(0; \mu; M_\psi(e, B))$ .

We have obtained now the anomalous contribution to the rescaled spinor partition function as a local  $\mathbf{P}$  gauge invariant polynomial in the fields  $e_\alpha{}^\gamma$  and  $B_\alpha{}^{\gamma\delta}$ , which also must be present in any classical gauge field dynamics consistent with renormalizability of the spinor partition function. The analysis of the corresponding scalar and vector field cases leads to results of the same general structure [45]. Hence, we are finally led to construct a minimal action for the gauge fields just in terms of these  $\mathbf{P}$  gauge invariant expressions.

For  $T \neq 0$  we restrict ourselves to the contributions of  $O(\partial^0, \partial^2)$  in the derivatives and obtain as minimal classical action to this order

$$\begin{aligned}
S_G(e, B) &= \int \det e^{-1} \left\{ \Lambda - \frac{1}{\kappa^2} \cdot R_{\alpha\beta}{}^{\alpha\beta} + \beta_1 \cdot T_{\gamma\alpha}{}^\gamma T_\delta{}^{\alpha\delta} \right. \\
&+ \beta_2 \cdot T_{\alpha\beta\gamma} T^{\alpha\beta\gamma} + \beta_3 \cdot T_{\alpha\beta\gamma} T^{\gamma\alpha\beta} + O(\partial^4) \left. \right\},
\end{aligned} \tag{57}$$

skipping possible total divergence terms. Here we have to introduce different couplings  $\kappa, \beta_1, \beta_2, \beta_3$  and the constant  $\Lambda$  which are independently renormalized by the one-loop contribution we determined above. Note that our reasoning automatically enforces a cosmological constant as to be expected from general renormalization considerations. The action Eq. (57) describes the classical gauge field dynamics correctly at sufficiently low momentum scales and small values of the couplings. Nevertheless, only a dynamics containing the huge number of different  $O(\partial^4)$  terms as well, coming along with the same number of independent couplings, will be consistent with renormalizability (see also [39]). Note that no terms of  $O(\partial^6)$  or higher are demanded by our reasoning.

If we set  $T = 0$  the minimal classical action must contain the terms

$$S_G(e) = \int \det e^{-1} \left\{ \Lambda - \frac{1}{\kappa^2} \cdot R_{\alpha\beta}{}^{\alpha\beta} + \alpha_1 \cdot R_{\alpha\beta}{}^{\alpha\beta} \cdot R_{\gamma\delta}{}^{\gamma\delta} \right. \\ \left. + \alpha_2 \cdot R_{\alpha\gamma}{}^{\alpha}{}_{\delta} \cdot R_{\beta}{}^{\gamma\beta\delta} + \alpha_3 \cdot R_{\alpha\beta\gamma\delta} \cdot R^{\alpha\beta\gamma\delta} \right\}, \quad (58)$$

if discarding total divergences. The couplings  $\kappa, \alpha_1, \alpha_2, \alpha_3$  and the constant  $\Lambda$  obtain again contributions from the one-loop scale anomaly which has been determined above. We emphasize that  $S_G$  is an action for gauge fields defined on Minkowski spacetime  $(\mathbb{R}^4, \eta)$  and is invariant on one hand under local  $\mathbf{P}$  gauge transformations, on the other hand under global Poincaré transformations reflecting the symmetries of the underlying spacetime.

Important aspects of the geometric version of the quantized theory (58) such as one-loop divergences and  $\beta$ -functions and its unitarity problems are discussed in [39] and references given there.

## 7. Conclusions

To disentangle the structure of spacetime from the description of gravity we have given a complementary conception of Poincaré symmetry as a purely inner symmetry. Its extension to local  $\mathbf{P}$  gauge symmetry has led us to introduce gauge fields defined on a fixed Minkowski spacetime. Their coupling to any other field has come out to be essentially fixed. We then constrained their dynamics imposing consistency with renormalization properties of a Dirac field in gauge field backgrounds. In an appropriate low energy limit the resulting gauge field action has been shown to reduce to a form yielding the same observational predictions as made in general relativity which confirms us to have obtained a sensible description of gravity within the present framework.

In our conception there is no direct interrelation between gravity and the structure of spacetime. Although it may be convenient to introduce a second, 'effective' metric on Minkowski spacetime to answer questions about the behaviour of rods and clocks in a classical context [20] we essentially deal here with a field theoretical description of gravitation free of any non-trivial geometrical aspects as proposed to investigate

in the introduction. Unfortunately, the resulting theory is in many aspects still too close to the geometric approach and is far from leading to a convincing quantization. This shows up most clearly in the necessity of including terms quadratic in the field strength in the classical gauge field action. Although the corresponding quantum theory is known to be renormalizable, the occurrence of negative energy or negative norm ghost states has destroyed up to now any attempt of establishing unitarity and hence a physical interpretation of the theory (see also [39]).

## 8. Acknowledgments

This work has partially been supported by Schweizerischer Nationalfonds. It is a pleasure to thank the organizers of the Erice school 1995 on Quantum Gravity for the warm and fruitful hospitality during the course. The discussions with F. W. Hehl in Erice and his friendly interest in this work were an important encouragement. I am indebted to C. Ford and F. Krahe for intensive discussions, to L. O’Raifeartaigh for clarifying some group theoretical aspects.

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