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Some Implications of Theoretical Physics for Epistemology

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As a theoretical physicist I welcome the stimulus provided by this Symposium to delve a little into epistemological problems which were always around but which multiplied considerably in the present century due largely to the advent of quantum mechanics. By way of introduction I shall recall some early investigations into quantum theory associated with the names of Heisenberg and Schrödinger, and the controversy between these two physicists regarding the interpretation of quantum mechanics (Moore, 1989).

We shall commence by mentioning some of the pioneering studies of the problem of finding a replacement for classical mechanics that would be valid for physical systems of atomic dimensions. Heisenberg was greatly influenced by a paper of Bohr, Kramers and Slater, in which it was proposed that an atom may be pictured as a set of oscillators with frequencies equal to its absorption frequencies. He set out to establish an atomic theory based entirely on observables like frequencies, amplitudes and polarizations of spectral lines. According to Balmer the frequency ν of the visible lines of the hydrogen spectrum is given by

$$\nu = Rc\left(\frac{1}{2^2} - \frac{1}{n^2}\right), \qquad (1)$$

where R is the Rydberg constant, c is the velocity of light _in vacuo_ and n is an integer greater than 2. Ritz generalized (1) to

$$\nu_{nm} = Rc\left(\frac{1}{m^2} - \frac{1}{n^2}\right) \qquad (2)$$

with

$$m = 1,2, \ldots; \; n = 2,3, \ldots \quad n>m.$$

Experiments by Ritz and Rydberg yielded the relation

$$h\nu_{nm} = E_n - E_m,$$

where h is the Planck constant, E_n is the initial and E_m is the final energy of the atom. Thus for a set of energy levels, which might be infinite, we can display the values of ν_{nm} as a matrix. When ν_{nm} is negative, it refers to absorption.

In analogy with (2) Heisenberg described the position q of an electron by a matrix with elements

$$q_{nm} \exp (2\pi i \nu_{nm} t)$$

and the momentum p of the electron by a matrix with elements

$$p_{nm} \exp (2\pi i \nu_{nm} t).$$

In collaboration with Born and Jordan he postulated that q and p are matrices satisfying (Heisenberg, Born and Jordan, 1925)

$$qp - pq = \frac{ih}{2\pi} \underset{\sim}{I},$$

where $\underset{\sim}{I}$ is the identity matrix. He also supposed that Hamilton's canonical equations

$$\dot{q} = \frac{\partial H}{\partial p}, \quad \dot{p} = -\frac{\partial H}{\partial q},$$

where H is the Hamiltonian of the system, are satisfied. Born, Heisenberg and Jordan made no attempt to construct a pictorial representation of what is happening in the atomic region when, for example, a spectral line is emitted. Their approach was that of the positivist school associated with the name of Ernst Mach, according to whom the sole purpose of scientific theory is to provide an economical way of recording observed experimental facts.

Mach's philosophy of science was not shared by Erwin Schrödinger, who in 1926 published a sequence of papers dealing with the emerging quantum mechanics (Schrödinger, 1928). In the first paper he derived a wave equation for the hydrogen atom from the Hamilton-Jacobi equation combined with the use of the calculus of variations and of certain boundary conditions. In the second paper

Schrödinger pointed out the analogy between classical mechanics and geometrical optics, and he suggested that there should exist an essentially classical theory of matter waves which would have the role in mechanics that Maxwell's theory of electromagnetic waves has in optics. This would require a particle of a mechanical system to be represented by a wave packet, namely, a group of waves with small dimensions in every direction. The same would be true for the image point of a mechanical system, that is, the point specified in multidimensional space by the generalized coordinates of the system. Thus Schrödinger appears to have thought that a classical wave picture based on continuous matter waves could provide a basis for atomic physics.

In the first paper Schrödinger obtained for a particle of mass m the wave equation, now known as Schrödinger's equation,

$$\nabla^2\psi + \frac{8\pi^2 m}{h^2}(E - V)\psi = 0,$$

where V is the potential of the particle, E is its total energy and ψ is the wave function. In terms of cartesian coordinates

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \quad ;$$

but for generalized coordinates q_1, q_2, $\cdots$ with twice the kinetic energy

$$2T = \sum_{i,k} b_{ik}\,\dot{q}_i\dot{q}_k \qquad (3)$$

∇^2 is a complicated function of the b_{ik}'s.

It was next shown by Schrödinger that his wave mechanics is equivalent to the Born-Heisenberg-Jordan matrix mechanics, if the matrix representatives of the dynamical variables are calculated with respect to the basis constituted by the normalized eigenfunctions of the Hamiltonian of the mechanical system and if, in accordance with the ideas of Louis de Broglie the linear momentum $\underline{p}$ is interpreted as $-ih/(2\pi)$ times the gradient operator. This result surprised Schrödinger very much, since his theory was based on a physical model while Heisenberg and his collaborators did not even admit the reasonableness of any such model.

Schrödinger postulated that a scientific theory should be based on a space-time continuum but he did not require that the continuum be that of the laboratory. In fact, if one were dealing with a two-electron problem, the i and k in (3) would assume the values 1,2,3,4,5,6 and so the continuum would be six-dimensional. On the other hand Heisenberg worked in a discontinuous space. Indeed he argued that discontinuity occurs everywhere in atomic physics, as is exemplified in any screen or Geiger counter experiment. Moreover he proposed that, when a transition can be made into several possible states, it is the observation of the physical process that makes the system jump into a particular state (Moore 1989, p. 452).

Bohr and Schrödinger became involved in a controversy regarding the interpretation of quantum mechanics. Schrödinger did not accept the notion of quantum jumps because he found it difficult to visualize them, and he rejected the notion that a transition between two states can occur instantaneously. Bohr agreed that quantum jumps cannot be visualized but he maintained that this does not prove that quantum jumps do not exist; on the contrary, the derivation of Planck's law of radiation requires that the energy of the atom have discrete values and that these values change discontinuously.

So far we have said little about the wave function itself. The interpretation of quantum mechanics was discussed again at the Solvay Conference held in Brussels during October 1927 and much of the discussion centred round the physical interpretation of ψ. Under the assumption that point particles exist within the atom Max Born had made the $\psi^*\psi$-hypothesis that the density of particles at any point within the atom is, apart from a normalizing factor, just $\psi^*\psi$. Schrödinger assumed that there exists within the atom not point particles but a charge and mass cloud and he made what he called the $\psi\psi^*$ - hypothesis, namely, that the cloud density at any point is given by a normalized $\psi\psi^*$. Born and Heisenberg disagreed strongly with the Schrödinger hypothesis and with Schrödinger's assertion that it is nonsense to talk about the trajectory of an electron within an atom. The views of Born and Heisenberg seem to have been generally accepted at the conference but Einstein, de Broglie and Schrödinger had strong reservations about them.

In a lecture entitled 'The Transformation of the Physical Concept of the World' and delivered at Munich in 1930 Schrödinger appeared to have moved his position somewhat (Erwin Schrödinger Collected Papers, 1984, Vol. 4, 600-608). This is significant, since Schrödinger usually showed himself reluctant to retreat from any position on which he had taken a stand. He described the two opposing schools of thought by saying that one postulates discontinuous quanta of energy and instantaneous jumps between energy states and that the other claims that matter consists of continuous waves filling all space. On account of the mathematical equivalence of matrix and wave mechanics mentioned earlier, each school is disinclined to condemn outright the views of the other one. Schrödinger accepts that waves, be they electromagnetic or matter-waves, are not to be considered as purely objective descriptions of physical reality but rather descriptions of the knowledge that we have of observations that have been made. Such observations disturb one another. This leads to an abandonment of any claim that science can provide a purely objective description of Nature.

In order to understand the import of this last assertion we recall that a matter of discussion among scientists and philosophers is the nature of the world of the scientist — the world which the scientist contemplates in the course of his professional activities and endeavours to interpret. Is the world picture purely subjective, is it purely objective, is it partly subjective and partly objective?

As an illustration of a world that is purely subjective we may recall some features of the thinking of Kepler (1571-1630) when he was seeking a description of the six planetary orbits known in his time (Heitler, 1963). His first attempt was to associate the orbits with the five regular solids, namely, the tetrahedron, cube, octohedron, dodecahedron, icosahedron. When this attempt failed, he tried to employ the notion of the disciples of Pythagoras that the motion of the planets is associated with musical sounds which make the heavens resound with harmonies. Since Pythagoras had discovered that different musical notes are associated with different lengths of vibrating strings, Kepler sought to link certain whole-number relationships with the planetary orbits. In the view of Kepler, God created the heavens as a perfect structure and the planetary system was created in order that the harmonies could resound in the heavens. It was found that there is a whole-number relationship between the angular velocities of a specified planet at its perihelion and its aphelion, and it was deduced that the system of six planets gives the whole major or minor scale depending on whether we start with the perihelion or aphelion of the planet Saturn.

In recent times the best known member of the subjective school was the late Sir Arthur Stanley Eddington who posed the question: What is the minimum set of observational data which is sufficient to form a basis for the whole edifice of physical theory? To understand Eddington's reply a distinction must be made between quantitative assertions like "The velocity of light *in vacuo* is 3×10^{10}cm. per second", "The value of the reciprocal of the fine-structure constant is 137", and qualitative assertions like "The velocity of light is independent of the motion of its source", "Inside a hollow electrified shell there is no electric field". Eddington's reply to the above question was framed in what is called Eddington's Principle: All the quantitative propositions of physics, that is, the exact values of the pure numbers that are constants of science, may be deduced by logical reasoning from qualitative assertions without making any use of quantitative data derived from observation (McConnell, 1958).

Among qualitative assertions Eddington included the postulates of impotence, for example, the uncertainty principle in quantum mechanics and the impossibility of constructing a perpetual motion machine: By applying his Principle Eddington proposed that the value of the reciprocal of the fine-structure constant is exactly the integer 137. He calculated the number N of particles in the universe and found it to be equal to the number of particles in Einstein's cylindrical model of the universe, if it is assumed that the ultimate constitutents of matter are electrons and protons. From N he deduced the ratio of the electrical to the gravitational force between two electrons. He also established a quadratic equation which gave the electron and proton masses to a high degree of accuracy.

It may be said that the subjective pictures of the worlds which we have described are unsatisfactory. Nowadays no reputable scientist would attempt to defend the Pythagorean theory on which Kepler based his cosmology, even though it led to the laws of planetary motion as a by-product. In Eddington's speculations a postulate of impotence like the impossiblity of constructing a perpetual motion machine is something that emerged as a result of countless experiments. Then the calculation of N by Eddington is no longer valid on account of the many elementary particles that have since been discovered and because Einstein's cylindrical model of the universe is no longer accepted.

We now consider pictures of the world that were accepted as purely objective. This means that no subjective features similar to those of Kepler and Eddington are introduced. The use of the purely objective picture was introduced by Galileo Galilei (1564-1642), who though a contemporary of Kepler had a very different outlook on the universe. According to Galileo scientific assertions must rely on observation and experience. A scientific hypothesis must be mathematically and logically sound, and it must lead to physical results which are in agreement with experiment. In this context one should be careful not to assume that agreement with experiment implies that a hypothesis is mathematically and logically sound. Indeed we have already mentioned that Kepler's laws of planetary motion were based on the discredited theory of harmonies in the universe. Moreover in the twentieth century Bohr obtained the Balmer formula (1) by employing the classical theory of Rutherford, according to which the electron in a hydrogen atom moves around the proton in a stable orbit. However such an electron would radiate energy continuously and the atom would quickly collapse.

The approach of Galileo was taken up generally by the scientific community and in particular by Sir Isaac Newton (1643-1727). Let us consider Newton's laws of motion. The force acting on a body at any time is equal to the mass of the body multiplied by the rate of increase of velocity. This determines the motion from one instant to the next. A similar situation occurs for the succeeding time interval. Hence, if we know the position and velocity of the body at any instant and the force at every instant of time, we can in principle determine the path in which the body moves and the velocity of the body at any point of its path. Thus Newtonian dynamics is deterministic.

Having discussed subjective and objective world pictures we return to the Munich lecture of Schrödinger. Though this lecture was delivered on 6th. May 1930, it remained unpublished during the lifetime of Schrödinger and it was not freely available until 1984 when it appeared in Vol. 4 of his collected papers. He wrote (Moore, 1989, pp. 250, 251):

> Our mind, by virtue of a certain finite, limited capability, is by no means capable of putting a question to Nature that permits a continuous series of answers. The observations, the individual results of measurements, are the answers of Nature to our discontinuous questionings. Therefore, perhaps in a very important way, they concern not the object alone, but rather the relations between subject and object ····· It is thus no longer so obvious that repetition of observations must lead ···· in the limit to an exact knowledge of the object. When we interpolate the actual measurements by the best possible means, they are embedded in continua ···· that do not represent the natural object in itself, but rather the relation between subject and object.
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> The different wave forms, the old long-familiar electromagnetic waves as well as the new so-called matter waves, are not to be considered as purely objective descriptions of reality ····· The wave functions do not describe Nature in itself, but the knowledge that we possess at any given time of the observations actually carried out. They allow us to predict the results of future observations not with certainty and precision but with just that degree of unsharpness and probability with which observations actually made on the object permit predictions about it. The wave

description that is presently accepted ... is based on the fact that observations mutually disturb one another — a circumstance that in one respect increases our knowledge of the object, in one respect decreases it.

Most of us today feel that this necessary abandonment of a purely objective description of Nature is a profound change in the physical concept of the world. We feel it as a painful limitation of our right to truth and clarity, that our symbols and formulas and the pictures connected with them do not represent an object independent of the observer but only the relation of subject to object. But is this relation not basically the one true reality that we know? Is it not sufficient that it finds a solid, clear, unequivocal expression, wherein in fact all truth exists? Why must we exclude ourselves completely?

Some twenty years after Schrödinger's Munich lecture the physical chemist and professional philosopher Michael Polanyi delivered a set of lectures in the University of Aberdeen which covered among other subjects the questions of subjectivity and objectivity in the acquisition of knowledge. In 1957 he published these lectures with some modifications as a book with the title Personal Knowledge: Towards a Post-Critical Philosophy.

In the course of an analysis of what scientists usually mean by 'objective' he writes (Polanyi 1957, p. 16):

> Modern man has set up as the ideal of knowledge the conception of natural science as a set of statements which is 'objective' in the sense that its substance is entirely determined by observation, even while its presentation may be shaped by convention. This conception, stemming from a craving rooted in the very depths of our culture, would be shattered if the intuition of rationality in nature had to be acknowledged as a justifiable and indeed essential part of scientific theory. That is why scientific theory is represented as a mere economical description of facts; or as embodying a conventional policy for drawing empirical inferences; or as a working hypothesis, suited to man's practical convenience — interpretations that all overlook the rational core of science.

Thus, according to Polanyi, many modern scientists follow the logical system called "positivism". We have already noted that this system was followed by the Copenhagen School when it was setting up matrix mechanics. On the other hand Schrödinger stood aside from positivism and endeavoured to present a rational picture of atomic processes.

Polanyi introduces prior belief as an element necessary for the acquisition of all knowledge (Polanyi, 1957, pp. 266-7). "Tacit assent and intellectual passions, the sharing of an idiom and of a cultural heritage, affiliation to a like-minded community: such are the impulses which shape our vision of the nature of things on which we rely for our mastery of things. No intelligence, however critical or original, can operate outside a fiduciary framework."

While our acceptance of this framework is the condition for having any knowledge, this matrix can claim no self-evidence ..... Science exists only to the extent to which there lives a passion for its beauty, a beauty believed to be universal and external ...... Our basic beliefs are indubitable only in the sense that we believe them to be so".

Polanyi concludes his argument with the words: This then is our

> liberation from objectivism: to realize that we can voice our ultimate convictions only from within our convictions —— from within the whole system of acceptances that are logically prior to any particular assertion of our own, prior to the holding of any particular piece of knowledge. If an ultimate logical level is to be attained and made explicit, this must be a declaration of my personal beliefs. I believe that the function of philosophic reflection consists in bringing to light, and affirming as my own, the beliefs implied in such of my thoughts and practices as I believe to be valid; that I must aim at discovering what I truly believe in and at formulating the convictions which I find myself holding; that I must conquer my self-doubt, so as to retain a firm hold on this programme of self-identification.

Since scientific theory, as we know it, came to us from philosophers of ancient Greece, it may be of interest to recall how the ancient Greeks looked at the description of Nature. The notion that reality has a rational structure expressible as scientific theory goes back to Thales of Miletus, who lived about 600 B.C., and to the school of philosophers which he founded. This notion implied that the world is intelligible. A feature of the scientific method which predates Thales and which has been generally accepted until the present century is that the intelligibility of the world is investigated as a reality belonging to the world and exterior to us. This approach simplified scientific discussion, the scientist as observer being excluded from the world picture that he is endeavouring to build. However from the time that Heisenberg proposed that the observation of a physical system makes it jump into a particular quantum state he ceased to exclude the scientist from the world picture. Heisenberg expresses his change of attitude as follows (Heisenberg pp. 28, 29):

> When we speak of the picture of nature in the exact science of our age, we do not mean a picture of nature so much as a picture of our

relationship with nature. The old division of the world into objective processes in space and time and the mind in which these processes are mirrored — in other words, the Cartesian difference between *res cogitans* and *res extensa* — is no longer a suitable starting point for our understanding of modern science. Science, we find, is now focused on the network of relationships between man and nature, on the framework which makes us as living beings dependent parts of nature, and which we as human beings have simultaneously made the object of our thoughts and actions. Science no longer confronts nature as an objective observer, but sees itself as an actor in this interplay between man and nature. The scientific method of analysing, explaining and classifying has become conscious of its limitations, which arise out of the fact that by its intervention science alters and refashions the object of investigation. In other words, method and object can no longer be separated. The scientific world-view has ceased to be a scientific view in the true sense of the word.

The consequences of Heisenberg's turning away from Machian philosophy have been elaborated by Heelan (1972) in the following manner:

This involved a conversion from the classical model of a subjectless scientific objectivity to the subject-dependent objectivity of quantum mechanics. Quantum mechanics arose as the outcome of Werner Heisenberg's reflection on the role of observables in science. By an "observable" he meant a quantity that, though not imaginable in a classical space-time model, was part of the interpretations of a mathematical model and was measurable in principle. His intuition rejected the objectivist presuppositions of classical physics and, in a profoundly significant epistemological shift, he consciously placed the measuring subject or observer at the heart of quantum mechanics. The classical physics of his time presupposed either no observer or one separated from matter and outside of history. The quantum-mechanical observer, on the other hand, is one of human scale who uses instruments of the same scale to observe quantum-mechanical events and processes. Quantum-mechanical observers, then, are as manifold as the kinds of instruments a scientist can use. The most significant discovery of quantum mechanics, however, is the fact that it is not possible to construct an instrument or a panel of instruments that will give simultaneously the values of all the observable properties of a quantum mechanical system. The most famous expression of this surprising discovery is Heisenberg's Uncertainty Principle, which relates the measure of inaccuracy (Δx) of a position measurement (x) with the associated measure of inaccuracy (Δp) of a momentum measurement (p) according to the inequality $\Delta x.\Delta p \geqslant h/(2\pi)$, where h is the Planck constant.

It is plain that the above developments in scientific thinking have serious implications for the traditional formulation of Applied Logic. Can there be in principle a strictly and fully logical representation of scientific knowledge? It seems clear that in the interplay between man and nature an understanding is humanly developed, which is in part a tacit mental activity, in part an explicit objective account. Until now the Logic of Science has been consciously objectivist, since it has had only to deal with science as a set of explicit statements. It is not equipped to deal with the tacit dimension of science as personal knowledge, and therefore is not fully equipped to apply itself to science viewed totally in both tacit and explicit dimensions. (Bastable, 1975, p. 387).

When theoretical physics is examined either as an activity of knowing or as a logical body of knowledge, it poses challenging questions to conventional attitudes towards science. Such questions do not raise doubts about the methodology of science. They do not divert the scientist from pursuing research in his special field or continuing to apply his own method of carrying out research. However the questions call for serious investigation, if the proper place of science in the context of human culture is to be identified.

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