DIAS-STP-86-01

FINITE TEMPERATURE QUANTUM ELECTRICAL NETWORK THEORY+

T. Garavaglia*

Institiuid Ard-Leighinn Bhaile Atha Cliath
Baile Atha Cliath 4
Eire(Ireland)

Introduction

Finite temperature field(FTF) theory provides an elegant method for describing thermal and quantum noise in an electrical network. This method is applied to give fluctuation dissipation theorem results for the second moments representing noise in a dissipative LRC quantum oscillator. Classical dissipation is understood from a phase space analysis. Quantum dissipation can be studied with the aid of an effective Lagrangian obtained from considering a semi-infinite low-pass filter. This provides a frequency cut-off which yields finite second moments for both charge and current. The method has been extended to interacting oscillators, coupled by mutual inductance, to investigate a system which may be useful in the detection of vibrations induced by gravitational radiation.

FTF Quantization of an Electrical Network

Extending methods from Refs. 2, and 3, the charge density field at inverse temperature $\beta = 1/KT$ is represented as a spectral integral

$$Q(x,y,\beta) = \int Q_\omega(x,y,\beta)d\omega. \tag{1}$$

This field along with its conjugate momentum satisfies the canonical commutation relation. These fields can be expanded in terms of the filter in-field operators

$$A^{in}(\beta)=(1+f(\beta))^{1/2}A(\omega,\beta)+f^{1/2}(\beta)\tilde{A}(\omega,\beta)\ , \quad f(\beta)=1/(e^{\beta\omega}-1), \tag{2}$$

which satisfy Boson commutation relations.

At frequency ω the Lagrangian density for a lumped circuit of inductances L_{ij} and capacitances C_{ij} is

$$\mathcal{L}(Q_\omega,\partial_t Q_\omega,\beta)=\delta(x)\sum_{i,j}(L_{ij}\partial_t Q_{\omega i}\partial_t Q_{\omega j} - C_{ij}Q_{\omega i}Q_{\omega j})/2$$

$$+ H(x)\sum_i L_{Ti}((\partial_t Q_{\omega i})^2 - v^2(\omega)(\partial_x Q_{\omega i})^2)/2.$$

+A longer version is available upon request.

*Also Institiuid Teicneolaiochta Bhaile Atha Cliath.

The part associated with the Heaviside distribution represents the effective Lagrangian for a low-pass filter of impedance

$$Z(a,b) = i\omega L_o/2 + (L_o/C_o - \omega^2 L_o{}^2/4)^{1/2} \tag{4}$$

which determines the velocity of propagation and the cut-off frequency.

The field equations are found from the action

$$S=\iiint (Q_\omega,\ \partial_t Q_\omega, \beta)\,dx\,dt\,d\omega. \tag{5}$$

Dissipative LRC Oscillator

Classical dissipation for an LRC oscillator of charge q and momentum $p=L\dot{q}$ is obtained in phase space from the modified Hamilton's equations

$$dQ/d\tau=\partial H/\partial P,\quad dP/d\tau = -\ \partial H/\partial Q - \partial(\tilde{\gamma}P^2/2)/\partial P \tag{6}$$

with $P=p/(\omega_o L)^{1/2}$, $Q=q/(\omega_o C)^{1/2}$, $\tilde{\gamma}=R/L\omega_o$, $\tau = t\omega_o$, $\omega_o=(LC)^{-1/2}$, and $2H=P^2+Q^2$. The phase space spirals are found from

$$dP/dQ+\tilde{\gamma}+P/Q = 0,\quad (P+aQ)^a/(P+bQ)^b = \text{Constant}. \tag{7}$$

with $a=\tilde{\gamma}/2+\Omega$, $b=\tilde{\gamma}/2-\Omega$, and $\Omega=((\tilde{\gamma}/2)^2-1)^{1/2}$.

Quantum dissipation is described by a spectral Langevin equation, obtained from (5), with a frequency dependent damping constant. The moments in terms of $z=2KT/\hbar\omega_o$ are found as matrix elements with finite temperature vacuum states to be

$$\sigma^2(Q,z) = \hbar K_1(Q_o,z)/L\omega_o 2K_2(Q_o,0) \tag{8a}$$

$$\sigma^2(L\dot{Q},z) = \hbar L\omega_o K_3(Q_o,z)/K_2(Q_o,0)2 \tag{8b}$$

where $Q_o(\nu) = Q_o/(1-(\nu/\Lambda)^2)^{1/2}$, $Q_o=L\omega_o/R, \nu = \omega/\omega_o$, $\Lambda=2Q_oC/C_o$ and where

$$K_m(Q_o,z) = \int_o^\Lambda d\nu\,\nu^m \coth(\nu/z)/\pi Q_o(\nu)((\nu^2-1)^2+(\nu/Q_o(\nu))^2)^2. \tag{8c}$$

The solutions are normalized so that $Q(t,\beta)$ and $L\dot{Q}(t,\beta)$ satisfy the Dirac bracket.

These methods have been extended to the case of interacting LRC oscillators which are coupled by mutual inductance. Expressions similar to (8) for the second moments of the separate branches of the circuit may be obtained in the fluctuation dissipation theorem form

$$\sigma^2(Q\ ,\beta) = (\hbar/2\pi)\int_o^\Lambda Z(z_1(\omega),\ z_2(\omega))\omega \coth(\hbar\omega/KT)d\omega. \tag{9}$$

Circuits of this type are being studied for their possible use in the detection of gravitational radiation.

References

1. T. Garavaglia, Finite Temperature Field Theory and Quantum Noise in an Electrical Network, DIAS STP-85-08 preprint: (1985).
2. H. Umezawa, H. Matsumoto, and M. Tachiki, "Thermo Field Dynamics and Condensed States," North-Holland, Amsterdam (1982).
3. B. Yurke, and J. S. Denker, Quantum Network Theory, Phys. Rev. A29: 1419 (1984).