Precision Test of Holographic Flavourdynamics

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Abstract We study the Berkooz-Douglas matrix model using holography, lattice simulation and high temperature perturbative expansion. In particular we calculate the mass susceptibility of the theory. Our results show excellent agreement between lattice simulations and holography at low and intermediate temperatures $T \leq \lambda^{1/3}$. We also report a surprisingly good agreement between holography and perturbative high temperature expansion at $T \sim \lambda^{1/3}$.

1 Introduction

Among the most profound developments of modern physics is the quantum description of reality. Quantum field theory (QFT) is our main tool to describe physics on a diverse range of scales, from the standard model of interactions to the theory of superconductivity. Yet there are regimes, when QFTs are strongly coupled and perturbation theory breaks down. These regimes are prevalent in Nature from confinement, chiral symmetry breaking and quark matter in particle physics to high temperature superconductors, strange metals and graphene in condensed matter phenomena. This calls for novel non-

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pertubrative tools to study QFTs at strong coupling. A promising such tool is the AdS/CFT correspondence [1, 2], also referred to as the holographic correspondence, which is a duality between a strongly coupled quantum field theory and a higher dimensional weakly interacting gravitational system. There is overwhelming evidence that the supersymmetric regime of the correspondence is correct, yet the most relevant phenomenological applications of the duality, when supersymmetry is broken are poorly tested, making the nature of these studies somewhat speculative. Performing highly non-trivial tests of the correspondence with emphasis on flavoured holographic gauge theories will be the subject of my research during my fellowship.

Testing the AdS/CFT correspondence requires an alternative nonperturbative approach and for a four dimensional gauge theory lattice simulations on a computer seem a natural approach. Unfortunately, although the subject of active research, the lattice formulation of four dimensional Supersymmetric Yang-Mills (SYM) theory is still problematic. When faced with such difficulties, a useful approach is to study simplified versions of the correspondence. Recently progress in this direction has been made by studying a 0+1 dimensional version of the correspondence, between the supersymmetric BFFS matrix model [3] and its dual type IIA supergravity background [4]-[10].

In this report we are interested in generalisation of the AdS/CFT correspondence [11] including matter in the fundamental representation of the gauge group. The idea of ref. [11] is to introduce a probe D7–brane to the $\mathrm{AdS}_5 \times S^5$ supergravity background. The corresponding dual field theory has $\mathcal{N} = 2$ supercharges and is the $\mathcal{N} = 4$ SYM theory coupled to an $\mathcal{N} = 2$ fundamental hypermultiplet, which is the effective low energy theory of the D3/D7 brane intersection. This holographic set-up has received a great deal of attention and has led to numerous theoretical and phenomenological applications. In particular at the finite temperature regime of the theory features a first order meson melting phase transition [12]-[17]

In ref. [18] the lattice formulation of the Berkooz-Douglas matrix model [19] (see also [20]) was studied. The main result of ref. [18] is a numerical calculation of the fundamental condensate of the theory using computer simulations. Comparison with holographic calculations show remarkable agreement in the deconfined phase of the theory. The most plausible explanation for that agreement is that in the deconfined phase the α' corrections due to the high curvature of the background are cancelled in the calculation of the condensate, since it involves a derivative of the free energy with respect to the bare mass of the theory. It is then natural to propose that this agreement should be even better if one considers the mass susceptibility of the condensate, which is a second derivative of the free energy with respect to the bare mass. In fact the mass susceptibility can be evaluated at zero bare mass, when analytic result for the susceptibility can be obtained from holography. In ref. [21] a perturbative approach (at high temperature) was used to calculate the condensate susceptibility, remarkably in ref. [22] it was shown that at intermediate temperature ($T \sim \lambda^{1/3}$) the holographic calculations agree with the perturbative high temperature expansion of ref. [21]. Furthermore, computer simulations of the lattice discretisation developed in ref. [18] show agreement at lower temperatures. Ref. [22] also considered an alternative lattice formulation providing an independent check of the numerical results.

The goal of this report is to discuss the results of refs. [18, 21, 22]. The structure of the paper is as follows:

In section 2 we describe the general properties of the Dp/Dq brane intersections T-dual to the D3/D7 system. We comment on the universal properties of the Dp/Dq system and the difficulties in simulating supersymmetric theories in higher than 1+ 0 dimensions. Section 3 describes the holographic calculation of the condensate susceptibility performed in ref. [22]. In section 4 we compare the holographic results with the results from field theory using both perturbative high temperature expansion [21] and lattice simulations [22]. Finally, section 5 contains a brief conclusion.

2 The Dp/Dq brane holographic set-up

The Dp/Dq holographic set-up is inspired by the Dp/Dq brane intersection T-dual to the D3/D7 one. In this set-up a probe Dq brane is introduced to the near horizon limit of the supergravity background describing a Dp-brane. In the dual field theory this corresponds to adding $\mathcal{N} = 2$ fundamental hypermultiplets in the language of $\mathcal{N} = 4$ four-dimensional SYM theory. The probe approximation corresponds to a quenched approximation in the dual field theory when fundamental loops are ignored in correlation functions involving only adjoint fields. In other words the dynamics of the adjoint degrees of freedom is not affected by the presence of the fundamental fields. Not that this is not the same as the quenched approximation in lattice gauge theory since the fermionic determinant is not suppressed when fundamental fields are present in the correlators (as in the case of the fundamental condensate).

In this set-up the asymptotic separation of the Dq–brane corresponds to the bare mass of the fundamental hypermultiplet and bending of the probe Dq-brane at infinity encodes the fundamental condensate. Furthermore, the spectrum of the semi-classical fluctuations of the probe correspond to the meson spectrum in the dual field theory. This allows one to use semi-classical calculations in supergravity to obtain non-perturbative quantum results for the dual field theory. In particular one can explore the phase structure of the dual theory at finite temperature and in the presence of various other control parameters. It turns out that the thermal properties of the Dp/Dq system exhibit some universal features [14] and particularly the pattern of the first order meson melting phase transition depends only on the dimension of the internal cycle wrapped by the probe Dq-brane in the transfer to the Dp–brane subspace. One can show that the D3/D7 system is in the class of universality as the D0/D4 system, which is dual to the Berkooz-Douglas matrix model and can be relatively easily simulated on a computer. Therefore, by performing a precision test of the holographic correspondence in the D0/D4 system we indirectly test properties of the D3/D7 system.

3 Holographic calculation of the condensate susceptibility

At low temperature the BD model is proposed to be dual to the D0/D4 holographic set-up[1]. The most understood case that we will focus on is the so called quenched approximation, when the flavour D4–branes are in the probe approximation [11]. In the near horizon limit the D0-brane supergravity background is given by:

$$ds^2 = -H^{-\frac{1}{2}}\, f\, dt^2 + H^{\frac{1}{2}} \left(\frac{du^2}{f} + u^2\, d\Omega_8^2 \right) \,,$$
$$e^{\Phi} = H^{\frac{3}{4}} \,, \qquad C_0 = H^{-1} \,, \tag{1}$$

where $H = (L/u)^7$ and $f(u) = 1 - (u_0/u)^7$. Here u_0 is the radius of the horizon related to the Hawking temperature via $T = 7/(4\,\pi\, L)\,(u_0/L)^{5/2}$ and the length scale L is given by $L^7 = 15/2\,(2\pi\alpha')^5\,\lambda$, with λ the 't Hooft coupling.

To introduce matter in the fundamental representation we consider the addition of N_f D4–probe branes. In the probe approximation $N_f \ll N$, their dynamics is governed by the Dirac-Born-Infeld action:

$$S_{\rm DBI} = -\frac{N_f}{(2\pi)^4\,\alpha'^{5/2}\,g_s} \int d^4\xi\, e^{-\Phi}\,\sqrt{-\det||G_{\alpha,\beta} + (2\pi\alpha')F_{\alpha,\beta}||}\,, \tag{2}$$

where $G_{\alpha,\beta}$ is the induced metric and $F_{\alpha,\beta}$ is the $U(1)$ gauge field of the D4–brane, which we will set to zero. Parametrising the unit S^8 in the metric (1) as:

$$d\Omega_8^2 = d\theta^2 + \cos^2\theta\, d\Omega_3^2 + \sin^2\theta\, d\Omega_4^2 \tag{3}$$

and taking a D4–brane embedding extended along: t, u, Ω_3 with a non-trivial profile $\theta(u)$, we obtain (after Wick rotation):

$$S^E_{\rm DBI} = \frac{N_f\,\beta}{8\,\pi^2\,\alpha'^{5/2}\,g_s} \int du\, u^3 \cos^3\theta(u)\,\sqrt{1 + u^2\, f(u)\,\theta'(u)^2}\,. \tag{4}$$

The embedding extremising the action (4) can be obtain by solving numerically the corresponding non-linear equation of motion. The AdS/CFT dic-

[1] The D0/D4 set-up belongs to a large class of Dp/Dp+4 -brane intersections exhibiting universal properties such as the presence of a meson melting phase transition. For more details look at refs. [12, 13, 14, 15, 17] as well as ref. [16] for an extensive review.

tionary then relates the behaviour of the solution at large radial distance u to the bare mass and condensate of the theory via [11], [14]:

$$\sin\theta = \frac{\tilde{m}}{\tilde{u}} + \frac{\tilde{c}}{\tilde{u}^3} + \dots \ , \tag{5}$$

where $\tilde{u} = u/u_0$ and the parameters $\tilde{m}$ and $\tilde{c}$ are proportional to the bare mass and condensate of the theory. Therefore, the mass susceptibility of the condensate at zero bare mass $\langle \mathcal{C}^m \rangle$ is proportional to:

$$\langle \mathcal{C}^m \rangle \propto -\left(\frac{d\tilde{c}}{d\tilde{m}}\right)\Big|_{\tilde{m}=0} = \frac{7\pi}{2} \frac{\csc(\pi/7)\, \Gamma(3/7)\, \Gamma(5/7)}{\Gamma(1/7)^2\, \Gamma(2/7)\, \Gamma(4/7)} \ . \tag{6}$$

The last expression was obtained by using that small $\tilde{m}$ implies small θ, and hence the equation of motion for θ can be linearised and solved analytically. Combining equation (6) with the exact expressions for the mass and condensate in terms of $\tilde{m}$ and $\tilde{c}$ [14, 18]:

$$\begin{aligned} m = m_q/\lambda^{1/3} &= \frac{u_0\, \tilde{m}}{2\pi\alpha'} = \left(\frac{120\, \pi^2}{49}\right)^{1/5} \left(\frac{T}{\lambda^{1/3}}\right)^{2/5} \tilde{m} \ , \\ \langle \mathcal{O}_m \rangle &= -\frac{N_f\, u_0^3}{2\, \pi\, g_s\, \alpha'^{3/2}}\, \tilde{c} = \left(\frac{2^4\, 15^3\, \pi^6}{7^6}\right)^{1/5} N_f\, N_c \left(\frac{T}{\lambda^{1/3}}\right)^{6/5} (-2\, \tilde{c}) \ , \end{aligned} \tag{7}$$

we obtain:

$$\begin{aligned} \langle \mathcal{C}^m \rangle &= 14^{1/5} 15^{2/5} \pi^{9/5} \left(\frac{\csc(\pi/7)\, \Gamma(3/7)\, \Gamma(5/7)}{\Gamma(1/7)^2\, \Gamma(2/7)\, \Gamma(4/7)}\right) N_f\, N_c \left(\frac{T}{\lambda^{1/3}}\right)^{4/5} \\ &\approx 1.136\, N_f\, N_c \left(\frac{T}{\lambda^{1/3}}\right)^{4/5} \ . \end{aligned} \tag{8}$$

Equation (8) is the holographic prediction for the mass susceptibility of the fundamental condensate, which in the next section we compare to the field theory results obtained by high temperature expansion and lattice simulations.

4 Field theory comparison

In figure 1 we present a comparison between the analytic epxression (8) obtained from holography and our field theory results. The red curve represents the holographic prediction (8), while the black dashed curve corresponds to the high temperature expansion curve:

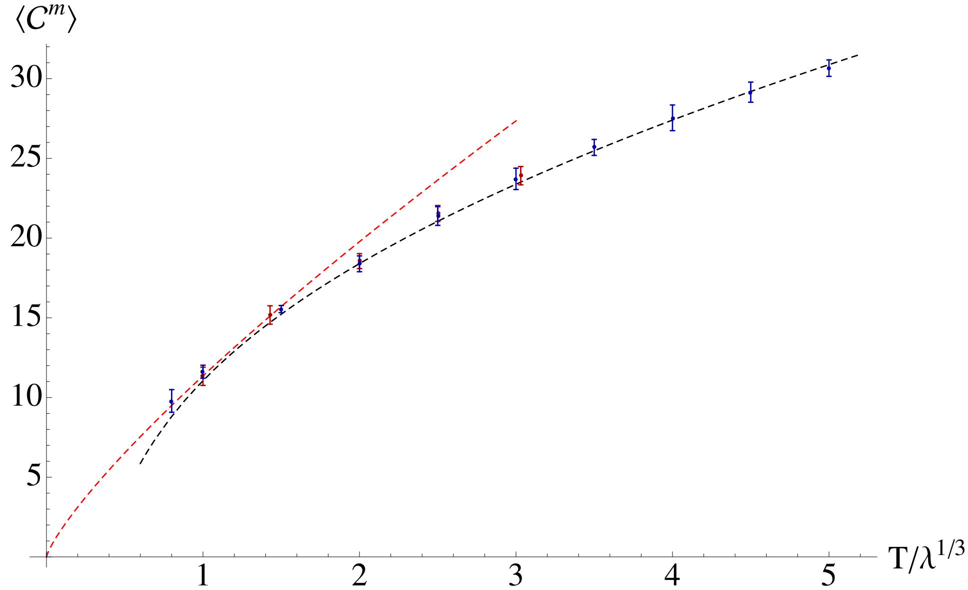

Fig. 1 The red curve represents the holographic prediction (8), while the black dashed curve corresponds to the high temperature expansion curve (9). The blue bars represent the results of lattice simulations using the lattice discretisation in ref. [18]. The red bars correspond to an independent lattice simulation based on an alternative formulation.

$$\langle \mathcal{C}^m \rangle = 14.08 \left(\frac{T}{\lambda^{1/3}} \right)^{1/2} - 3.02 \left(\frac{T}{\lambda^{1/3}} \right)^{-1} + O(T^{-\frac{5}{2}}) \ , \tag{9}$$

obtained in ref. [21]. The blue bars represent the results of lattice simulations using the lattice discretisation in ref. [18]. The red bars correspond to independent lattice simulations based on a different lattice discretisation.

Overall, one can observe excellent agreement of the lattice simulation and the high T curve even for temperatures as low as $T = \lambda^{1/3}$. One can also observe excellent agreement with holographic predictions at temperatures $T \sim \lambda^{1/3}$. Remarkably, even the high temperature curve is very close to the holographic curve in this regime. As mentioned earlier this suggests that the α' corrections to the mass susceptibility are indeed very small.

5 Conclusion

In this paper we reported on a recent study of the Berkooz-Douglas matrix model using both holography and field theory approaches. We focus on the study of the mass susceptibility of the condensate, for which we derive an analytic expression from holography. Since the curvature of the D0 supergravity background grows with the radial distance, significant α' corrections are expected at large and intermediate temperature (radious of the black hole). Naively one would expect that the holographic result for the susceptibility should be valid only at low temperature ($T < \lambda^{1/3}$). However, as argued in ref. [18], in the deconfined phase of the theory the derivatives of the free energy with respect to the bare mass should largely cancel the curvature α' corrections. Therefore, one can expect a good agreement even at intermediate temperature ($T \sim \lambda^{1/3}$). Remarkably, this is exactly what we observe in section 4, where not only lattice simulation agree with the theoretical curve (8), but also the high temperature expansion curve (9) is very close to the holographic one at $T \sim \lambda$. Overall our results provide a solid evidence for the validity of the holographic description of the Berkooz-Douglas model, even when supersymmetry is broken by a finite temperature.

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