Asymptotically flat, stable black hole solutions in Einstein–Yang-Mills–Chern-Simons theory

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Abstract

We construct finite mass, asymptotically flat black hole solutions in $d = 5$ Einstein–Yang-Mills–Chern-Simons theory. Our results indicate the existence of a second order phase transition between Reissner-Nordström solutions and the non-Abelian black holes which generically are thermodynamically preferred. Some of the non-Abelian configurations are also stable under linear, spherically symmetric perturbations. In addition a solution in closed form describing an extremal black hole with non-Abelian hair is found for a special value of the Chern-Simons coupling constant.

Introduction.– The so called "no-hair" conjecture [1] states that an asymptotically flat, stationary black hole is uniquely described in terms of a small set of asymptotically measurable quantities. However, in recent years counterexamples to this conjecture were found in several theories, most of them containing non-Abelian matter fields. The first non-Abelian "hairy" black hole solutions within the framework of $d = 4$ SU(2) Einstein-Yang-Mills (EYM) theory, were presented in [2]. Although these solutions were static and spherically symmetric with vanishing Yang-Mills (YM) charges, they were different from the Schwarzschild black hole and, therefore, not characterized exclusively by their total mass. However, all known asymptotically flat EYM solutions are perturbatively unstable and thus they do not contradict the spirit of the "no-hair" conjecture (see the review [3]). When considering instead a number $d > 4$ of spacetime dimensions, no finite energy asymptotically flat non-Abelian solutions are found [4, 5] unless the action is supplemented with string-theory inspired higher order YM curvature terms [6], in which case, again the solutions are classically unstable.

In odd spacetime dimensions, the usual gauge field action can be augmented instead by a Chern-Simons (CS) term. Such (CS) terms appear in various supersymmetric theories. In the Abelian case this leads to some new features for rotating black holes only [7]. In the non-Abelian case however, the CS term can affect the properties of solutions even in the static, spherically symmetric case. For the $d = 5$ case considered here, this allows the construction of finite mass, asymptotically flat, non-Abelian black hole solutions which generically turn out to be thermodynamically favoured over the Reissner-Nordström Abelian configurations. Moreover, some of these solutions are stable against linear, spherically symmetric perturbations.

The model.– In a $4+1$ dimensional spacetime, the smallest simple gauge group supporting a nonvanishing CS term is $SO(6)$. Then we consider a general EYMCS theory with Lagrangian

$$\mathcal{L} = \frac{1}{16\pi G} R *\mathbb{1} - \frac{1}{4} *F \wedge F - \kappa \epsilon_{I_1 \cdots I_6} \Big(F^{I_1 I_2} \wedge F^{I_3 I_4} \wedge A^{I_5 I_6} - g F^{I_1 I_2} \wedge A^{I_3 I_4} \wedge A^{I_5 J} \wedge A^{J I_6} + \frac{2}{5} g^2 A^{I_1 I_2} \wedge A^{I_3 J} \wedge A^{J I_4} \wedge A^{I_5 K} \wedge A^{K I_6} \Big), \tag{1}$$

where A^{IJ} are the $SO(6)$ gauge fields, $F^{IJ} = dA^{IJ} + gA^{IK} \wedge A^{KJ}$, G is gravitational constant, κ the CS coefficient and g the gauge coupling constant.

Our solutions are spherically symmetric with a line element

$$ds^2 = -f_0 dt^2 + f_1 dr^2 + f_2 d\Omega_3^2, \tag{2}$$

where f_i are functions of r and t in general and $d\Omega_3^2 = d\psi^2 + \sin^2\psi(d\theta^2 + \sin^2\theta d\phi^2)$ is the line element of the three dimensional sphere.

The general spherically symmetric Ansatz for the $SO(6)$ YM field is given [8]. In the present work we restrict ourselves to a consistent truncation, $SO(4) \times SO(2)$, of that Ansatz. Apart from simplifying the picture, prominent new physical features are conveniently exposed in this truncation of the Ansatz, stated as,

$$A = \frac{1}{g} \left(\frac{w(r,t)+1}{r} \Sigma_{ij} \frac{x^i}{r} dx^j + V(r,t) \Sigma_{56} dt \right), \quad \text{with} \quad i,j = 1, \ldots, 4, \tag{3}$$

Σ_{ij} being the representation matrices of $SO(4)$, and Σ_{56} of the $SO(2)$, subalgebras of $SO(6)$. The Cartesian coordinates x^i are related to the spherical coordinates (r, ψ, θ, ϕ) as in flat space.

Starting with static solutions, we have found it convenient to choose for the metric function in (2), $f_1 = 1/N(r)$, $f_2 = r^2$, $f_0 = N(r)\sigma^2(r)$, with $N(r) = 1 - m(r)/r^2$. Then the coupled static EYMCS equations of motion reduce to

$$m' = \frac{1}{2}\alpha^2 \left(3r \Big(N w'^2 + \frac{(w^2-1)^2}{r^2} \Big) + \frac{r^3}{\sigma^2} V'^2 \right), \quad \frac{\sigma'}{\sigma} = \frac{3\alpha^2 w'^2}{2r},$$
$$(r\sigma N w')' = \frac{2\sigma w(w^2-1)}{r} + 8\kappa V'(w^2-1), \quad (\frac{r^3 V'}{\sigma})' = 24\kappa(w^2-1)w', \tag{4}$$

where the prime indicates the derivative with respect to r and $\alpha^2 = 16\pi G/(3g^2)$ (a factor of $1/g$ is also absorbed in κ). The last equation above has the first integral

$$V' = \frac{\sigma}{r^3}(K + 8\kappa w(w^2-3)), \tag{5}$$

with K an integration constant.

The set of equations (4) admit the scaling transformation

$$r \to \lambda r, \quad m \to \lambda^2 m, \quad \sigma \to \sigma, \quad w \to w, \quad V \to V/\lambda, \quad \kappa \to \lambda\kappa \quad \text{and} \quad \alpha \to \lambda\alpha, \tag{6}$$

which is used in what follows to set $\alpha = 1$.

We consider EYMCS black holes, with an event horizon located at $r = r_h > 0$. In the vicinity of the event horizon, one finds the following expansion of the solutions

$$m(r) = r_h^2 + m_1(r - r_h) + \dots, \ \sigma(r) = \sigma_h + \frac{3\sigma_h w_1^2}{2r_h}(r - r_h) + \dots,$$
$$w(r) = w_h + w_1(r - r_h) + \dots, \ V(r) = v_1(r - r_h) + \dots, \tag{7}$$

where $v_1 = \frac{\sigma_h(K+8\kappa(w_h^2-3))}{r_h^3}$, $m_1 = \frac{r_h^4 v_1^2+3\sigma_h^2(1-w_h^2)^2}{2r_h\sigma_h^2}$, $w_1 = \frac{2(4\kappa r_h v_1+\sigma_h w_h)(1-w_h^2)}{(m_1-2r_h)\sigma_h}$. while for large values of r, the expression of the solutions is

$$m(r) = M - \frac{Q^2}{r^2} + \dots, \quad \sigma(r) = 1 - \frac{J^2}{r^6} + \dots, \quad w(r) = \pm 1 + \frac{J}{r^2} + \dots, \quad V(r) = V_0 - \frac{Q}{r^2} + \dots \ . \tag{8}$$

In the above relations, σ_h, w_h and J, M, V_0 are parameters given by numerics which fix all higher order terms, while $Q = K/2 \mp 8\kappa$.

The only conserved quantities associated with these solutions are the mass $\mathcal{M} = \frac{3\pi}{8G}M$ and the electric charge $\mathcal{Q} = \frac{4\pi^2}{g}Q$, which is associated with the $U(1)$ gauge symmetry generated by Σ_{56}. Other quantities of interest are the chemical potential $\Phi = \frac{V_0}{g}$, the Hawking temperature $T_H = \frac{1}{4\pi}\sigma(r_h)N'(r_h)$ and the entropy $S = \frac{\pi^2 r_h^3}{2G}$.

The solutions.– The moduli space of black hole solutions includes the Reissner-Nordström black hole (hereafter RN), described by $m(r) = M - \frac{Q^2}{r^2}$, $V(r) = V_0 - Q/r^2$, $\sigma(r) = 1$, $w(r) = \pm 1$. This solution has an event horizon at $r_h = \left(\frac{M}{2} + \sqrt{\frac{M^2}{4} - Q^2}\right)^{1/2}$, which becomes extremal for $Q = \frac{M}{2}$. We have found that for $\kappa \geq 1/8$ and a given $Q > 0$, the RN black hole presents an instability with respect to static non-Abelian perturbations, for a critical value of the mass. This instability is found within the Ansatz (3), for values of the magnetic gauge potential $w(r)$ close to the value -1 everywhere, $w(r) = -1 + \epsilon W(r)$. The perturbation $W(r)$ starts from some nonzero value at the horizon and vanishes at infinity, being a solution of the linear equation

$$r(rNW')' - 4(1 - \frac{8\kappa Q}{r^2})W = 0 \ , \tag{9}$$

where $N = 1 - \frac{Q^2+r_h^4}{r_h^2 r^2} + \frac{Q^2}{r^4}$. The second term in (9) shows the existence of an effective mass term μ^2 for W near the horizon, with $\mu^2 \sim 1 - 8\kappa Q/r_h^2$. All physical solutions have $\mu^2 < 0$, with $Q/r_h^2 = U(\kappa)$ being a monotonic function of the CS coupling constant κ. Then, for given $\kappa \geq 1/8$, an instability occurs for a critical value of the mass to charge ratio of the RN solution, with $M/Q = (1 + U^2)/U$. As $\kappa \to 1/8$, one finds $U = 1$, while $U \simeq 1/4\kappa$ for large κ. No solutions of (9) are found for $\kappa < 1/8$, or for perturbations of the form $w(r) = +1 + \epsilon W(r)$, in which cases the effective mass for W is always real.

This unstability signals the presence of a symmetry breaking branch of non-Abelian solutions bifurcating from the RN black hole. These nonperturbative configurations are found by solving numerically the Eqs. (4), using a shooting method. In this work we have restricted attention

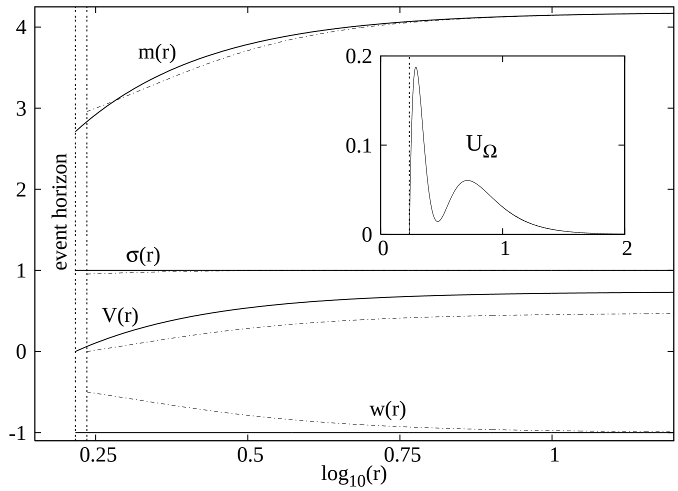

Figure 1: The profiles of a typical non-Abelian (dotted line) solution with $\kappa = 0.5$ is shown together with the corresponding Reissner-Nordström (RN) black hole (solid line) with the same mass and electric charge parameters $M = 4.186$ and $Q = 2$, respectively (the horizon radius of RN is smaller than that of the non-Abelian black hole). The intlet shows the corresponding potential for the perturbation equation (11) below.

to solutions with a monotonic behaviour[1] of the magnetic gauge potential $w(r)$. In contrast to other asymptotically flat non-Abelian black holes [2], some of the EYMCS solutions have no nodes in the magnetic gauge function $w(r)$. A typical nodeless profile is shown in Fig. 1, together with the corresponding Abelian solution with the same mass and electric charge.

Solutions smoothly interpolating between the asymptotics (7) and (8) appear to exist for any value of the CS coupling constant $\kappa \geq 1/8$. In a canonical ensemble, the non-Abelian black holes with a given κ exist for a finite interval of r_h (*i.e.* of the entropy) only. The detailed picture depends however on the ratio Q/κ, with a critical value $Q^{(c)} = 16\kappa$. For $Q \neq Q^{(c)}$, the temperature reaches its maximum at some intermediate value of the event horizon radius. Then a plot of the horizon area as a function of the temperature reveals the existence of several branches of non-Abelian solutions. The upper branch ends in the critical RN solution with $r_h = \sqrt{Q/U(\kappa)}$. The lower branch possesses always a positive specific heat, the Hawking temperature vanishing there for a minimal value $r_h^{(min)}$ of the event horizon radius. As $r \to r_h^{(min)}$, an extremal non-Abelian black hole solution with a regular horizon is approached. For $Q < Q^{(c)}$ the near horizon expansion of the solutions implies $r_h^{(min)} = 4\sqrt{3}\kappa(1 - w_h^2)/\sqrt{64\kappa^2 - w_h^2}$, where w_h satisfies the equation $(64\kappa^2 - w_h^2)Q^2 + 2\kappa(1 + w_h)^2(128\kappa^2(w_h - 2) + w_h(w_h^2 - 2w_h + 3)) = 0$; for $Q > Q^{(c)}$, the limiting solution has $r_h^{(min)} = \sqrt{Q - 16\kappa}$ and $w_h = 1$. Some of these features are shown in Fig. 2 where we plot the reduced area of the horizon $a_H = 2\pi^2 r_h^3/Q^{3/2}$ as a function of the dimensionless temperature t_H for a fixed CS coupling constant and several values of

[1]Although there exist solutions where w has local extrema, it is likely that these are always thermodynamically disfavoured because spatial oscillations in w increase the total mass.

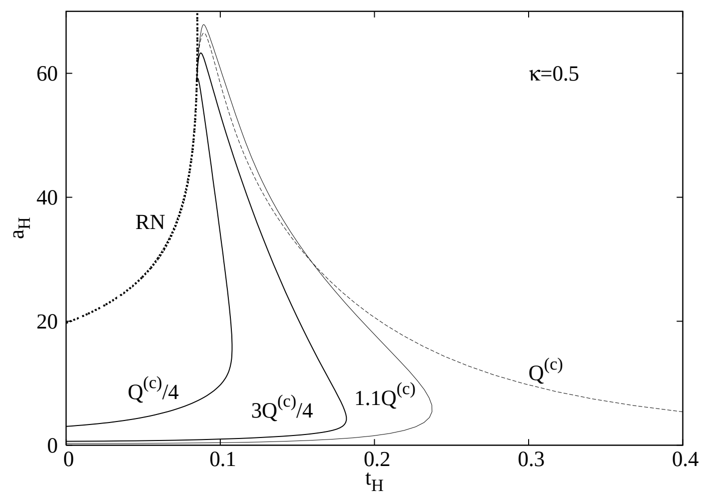

Figure 2: The dimensionless horizon area is plotted *vs.* the scaled temperature for the non-Abelian solutions with several values of the electric charge and a given value of κ. The branch of Reissner-Nordström solutions is also shown.

the charge parameter Q (both a_H and t_H are invariant under the scaling symmetry (6)). The branch of RN solutions is also shown there.

Furthermore, it turns out that the free energy $F = \mathcal{M} - T_H S$ of a RN solution is larger than the free energy of a lower branch non-Abelian solution with the same temperature and electric charge, except for configurations with κ close to 1/8 and small enough values of the charge, $Q \lesssim Q^{(c)}/3$. Therefore the non-Abelian black holes are generically preferred[2]. This reveals the existence of a second order phase transition between the RN solutions and the non-Abelian solutions. These aspects are exhibited in Fig. 3 where the dimensionless free energy $f = GF/Q$ is plotted as a function of the dimensionless temperature $t_H = T_H\sqrt{Q}$ for two values of κ. Moreover, as seen *e.g.* in Fig. 1, for the same values of the mass and electric charge, the RN solution typically has a smaller event horizon radius (and thus a smaller entropy), than the non-Abelian black hole. The parameter J which enters the large r asymptotics of the magnetic gauge potential $w(r)$ increases from zero (for the critical RN solution) to a maximal value approached at $T_H = 0$.

The overall picture is somehow different for $Q = Q^{(c)}$, in which case, despite the presence of an electric charge, the non-Abelian black holes behave in a similar way to the vacuum Schwarzschild-Tangherlini solution, with a single branch of thermally unstable configurations, see Fig. 2. In the limit $r_h \to 0$, these black holes approach a set of globally regular particle-like solutions, whose mass is an almost linear function of κ.

In the special case $\kappa = 1/8$, following the approach in [9], one finds the following exact

[2]Note that the non-Abelian solutions with large enough temperature have no RN counterparts, see Fig. 3.

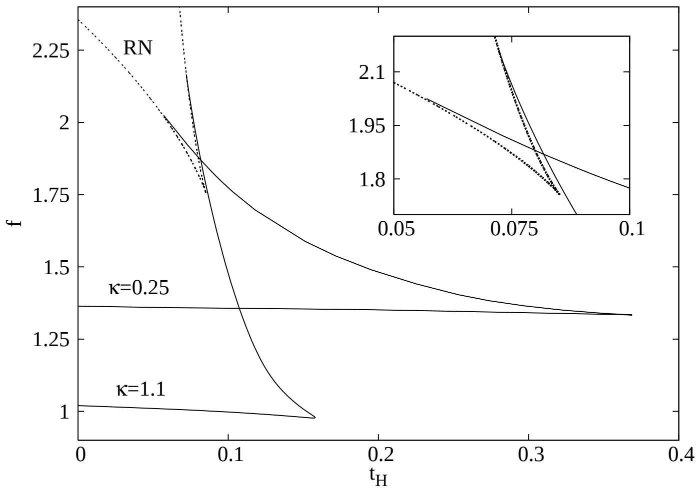

Figure 3: The scaled free energy is plotted *vs.* the scaled temperature for the Reissner-Nordström solutions (dotted line) and the two non-Abelian sets of solutions with different values of the Chern-Simons coupling constant κ and the same charge parameter $Q = 3.8$.

solution of the EYMCS equations within the general Ansatz (2), (3):

$$w = \frac{J - 2r^2}{J + 2r^2}, \quad V = F, \quad f_0 = F^2, \quad f_1 = \frac{f_2}{r^2} = \frac{1}{F}, \quad \text{with} \quad F = \left(1 + \frac{Q-2}{r^2} + \frac{2(r^2 + J)}{(r^2 + J/2)^2}\right)^{-1}, \tag{10}$$

where $Q \geq 2$ and J are arbitrary parameters. This describes an extremal black hole with non-Abelian hair, the regular event horizon being at $r = 0$ (in these coordinates). The mass, electric charge and entropy of this non-Abelian deformation of the extremal RN solution are $\mathcal{M} = \mathcal{Q}/g = \frac{4\pi^2}{g^2}Q$, $S = 8\pi^3(Q-2)^{3/2}/3g^2$ (note that J does not enter any physical quantity). In the limit $Q \to 2$, the solution (10) describes a particle-like soliton with a regular origin.

The stability of solutions.– The fact that we have found nodeless solutions suggest the existence of non-Abelian configurations stable against spherically symmetric perturbations. In examining such time-dependent fluctuations, we consider the metric Ansatz (2) with $f_1 = 1 - \frac{m(r,t)}{r^2}$, $f_2 = r^2$, $f_0 = (1 - \frac{m(r,t)}{r^2})\sigma^2(r,t)$, the YM Ansatz (3) and the following perturbed variables[3]

$$m(r,t) = m(r) + \epsilon m_1(r) e^{i\Omega t}, \ \sigma(r,t) = \sigma(r)(1 + \epsilon\sigma_1(r) e^{i\Omega t}),$$
$$w(r,t) = w(r) + \epsilon w_1(r) e^{i\Omega t}, \ V(r,t) = V(r) + \epsilon V_1(r) e^{i\Omega t}.$$

One finds that in the system of linearised EYMCS equations, the functions m_1, V_1 and σ_1 can be eliminated in favor of $w_1(r)$, leading to a single Schrödinger equation for w_1:

$$-\frac{d^2\chi}{d\rho^2} + U_\Omega(\rho)\chi = \Omega^2\chi, \tag{11}$$

[3]The corresponding problem for the SU(2) EYM system has been considered in [5].

where $\chi = w_1\sqrt{r}$, $dr/d\rho = N\sigma$ and

$$U_\Omega = \frac{N\sigma^2}{r^2}\Big[6(w^2 - w'^2 - \frac{1}{6}) - \frac{5N}{4} + 12(w^2-1)\frac{ww'}{r} + \frac{(1-w^2)^2}{r^2}(\frac{9}{2}w'^2 + 192\kappa^2 - \frac{3}{4}) + \frac{16\kappa V'}{\sigma}(rw - 3(1-w^2)w') - \frac{r^2V'^2(1-6w'^2)}{4\sigma^2}\Big]. \quad (12)$$

The potential above is regular in the entire range $-\infty < \rho < \infty$. Near the event horizon, one finds $U_\Omega \to 0$; for large values of ρ the potential is positive and bounded. Standard results from quantum mechanics [10] further imply that there are no negative eigenvalues for Ω^2 (and then no unstable modes) if the potential U_Ω is everywhere positive.

Although the potential (12) is not positive definite for all values of Q, κ, we have found numerically that the condition $U_\Omega > 0$ is fulfilled by some of the nodeless solutions (see the Fig. 1 for a such a configuration). Therefore at least some of our solutions are linearly stable. The full picture is, however, quite complicated and a detailed discussion will be presented elsewhere. We note only that, by using the approach in [11] we have found that the EYMCS solutions with one node in $w(r)$ we have constructed are indeed unstable.

Further remarks.– The main purpose of this work was to provide an example of stable, non-Abelian black holes without scalar fields, in asymptotically Minkowski spacetime.

One should note that, despite the different asymptotic structure of spacetime and the different horizon topology, the solutions in this work have some similarities with the colorful black holes with charge in Anti-de Sitter (AdS) space [12, 13], which provide a model of holographic superconductors. In both cases, an Abelian gauge symmetry is spontaneously broken near a black hole horizon with the appearance of a condensate of non-Abelian gauge fields there, leading to a second order phase transition. It remains an interesting open problem to clarify if the asymptotically flat EYMCS black holes may also provide useful analogies to phenomena observed in condensed matter physics. However, given the presence of several branches, the picture is more complicated for asymptotically flat solutions and we could not find so far simple universal relations between the relevant parameters as those found in [12] in the AdS case.

The black holes in this work admit a straightforward generalisation with a cosmological constant. For $\Lambda < 0$, the basic properties of asymptotically AdS configurations were discussed in [8]. In this case the EYMCS model with a special value of κ can be thought of as a truncation of the $\mathcal{N} = 8$, $d = 5$ gauged supergravity [14, 15], with all scalars there taking constant values. We have found that most of the properties of the asymptotically flat solutions, in particular the existence of stable configurations, hold also in the AdS case. In this case the solutions within the restricted $SO(4) \times O(2)$ Ansatz (3) contain already all relevant features of the full $SO(6)$ configurations.

While the existence of hairy non-Abelian black holes with AdS asymptotics is not a surprise in view of the results in [5], [16], we have found that, the $\Lambda = 0$ solutions presented here admit generalisations with de-Sitter asymptotics ($\Lambda > 0$) as well. These black holes, possesing a regular cosmological event horizon at $r = r_c > r_h$, were constructed within the same Ansatz as in the asymptotically flat case. By using an inflationary coordinate system, one finds also the following generalisation of (10), with $\kappa = 1/8$ again and

$$ds^2 = \frac{e^{2Ht}}{F(r,t)}(dr^2 + r^2 d\Omega_3^2) - F^2(r,t)dt^2,\ w(r) = \frac{J - 2r^2}{J + 2r^2},\ V(r,t) = F(r,t), \tag{13}$$

where $F(r,t)^{-1} = 1 + e^{-2Ht}\left(\frac{Q-2}{r^2} + \frac{2(r^2+J)}{(r^2+J/2)^2}\right)$ and $\Lambda = 6H^2$. This configuration describes a non-Abelian deformation of the extremal RN-de Sitter black holes in [17]. By using the methods in [17], [18], one finds that (13) shares the basic features of the $J = 0$ Abelian configuration.

Finally, we conjecture that similar EYMCS solutions exist in higher (odd) dimensions, the features encountered here being universal.

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