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Generalized baryon form factors and proton structure functions in the Sakai-Sugimoto model

C. A. Ballon Bayona [a]
Henrique Boschi-Filho [b], Nelson R. F. Braga [b], Matthias Ihl [c]
Marcus A. C. Torres [b]

[a] *Centre for Particle Theory,*
University of Durham,
Science Laboratories, South Road,
Durham DH1 3LE, United Kingdom.
`c.a.m.ballonbayona@durham.ac.uk`

[b] *Instituto de Física,*
Universidade Federal do Rio de Janeiro,
Caixa Postal 68528,
21941-972 Rio de Janeiro, RJ, Brasil.
`boschi@if.ufrj.br`,
`braga@if.ufrj.br`,
`mtorres@if.ufrj.br`

[c] *School of Theoretical Physics,*
Dublin Institute for Advanced Studies,
10 Burlington Rd,
Dublin 4, Ireland.
`msihl@stp.dias.ie`

Abstract

We investigate the production of positive parity baryon resonances in proton electromagnetic scattering within the Sakai-Sugimoto model. The latter is a string model for the non-perturbative regime of large N_c QCD. Using holographic techniques we calculate the generalized Dirac and Pauli form factors that describe resonance production. We use these results to estimate the contribution of resonance production to the proton structure functions. Interestingly, we find an approximate Callan-Gross relation for the structure functions in a regime of intermediate values of the Bjorken variable.

Contents

1. Introduction

In the regime of low momentum transfer ($\sqrt{q^2}$ lower than a few $GeVs$), non-perturbative effects become relevant in hadronic scattering. Lattice QCD (the numerical approach of substituting the continuous spacetime by a lattice of points) is quite difficult to use here because of the real time dependence in scattering processes. Effective models can be very useful but they have a low predictability because they may depend on many parameters.

In recent years, gauge/string duality has evolved into an invaluable tool to study many strongly coupled phenomena in particle and condensed matter physics. In particular, it has brought a new insight into the strongly coupled regime of Yang-Mills theories. A good example is the successful prediction of the shear viscosity for strongly coupled theories whose gravity dual involves a black hole in Anti-de-Sitter space [1]. This prediction is in accordance with the small value observed in the quark-gluon plasma phase observed in heavy ion collisions at RHIC and LHC. The study of 10-d string models (top-down approach) and 5-d effective models (bottom-up approach) dual to 4-d QCD-like theories is known as AdS/QCD (see [2,3,4,5] for a review of the top-down approach, [6,7,8] for the bottom-up approach and [9] for an hybrid approach). AdS/QCD is a powerful approach to the strong coupling regime of QCD because the string models depend on very few parameters (the string length, string coupling and number of colors and flavours of the dual theory). On top of that, general properties of hadron phenomenology (like vector meson dominance) seem to be universal in this approach in the sense that they do not depend on the particular AdS/QCD model. There is one string model, though, that has a field content very similar to large N_c QCD in the regime of large distances and massless quarks. This is the Sakai-Sugimoto model [10,11] that realizes confinement and chiral symmetry breaking.

In this paper we investigate the production of baryon resonances with positive parity in proton electromagnetic scattering within the Sakai-Sugimoto model. First we define the current matrix element that describe the transition from a proton to a positive parity baryonic resonance. We derive the general expansion of the current matrix element in terms of scalars that we define as generalized Dirac and Pauli form factors which include as a particular case the well known elastic Dirac and Pauli form factors. Using holographic techniques we obtain a relation between the generalized Dirac and Pauli form factors and the couplings between baryons and vector mesons. This result can be interpreted as a realization of vector meson dominance in electromagnetic scattering of baryons. We use our results for the generalized Dirac and Pauli form factors to estimate the helicity amplitudes that describe transitions between baryons with different helicities.

We also estimate in this paper the contribution of resonance production to the proton structure functions. The latter are Lorentz invariant scalars defined in Deep Inelastic Scattering (DIS) which is the inclusive scattering of a proton by a virtual photon (emitted by a lepton). The proton structure functions have a nice interpretation in terms of parton distribution functions that describe probability densities of finding partons (valence quarks, gluons and quark-antiquark pairs) with a fraction of the longitudinal momentum of the proton. A calculation of these quantities at low momentum transfer proves difficult in non-perturbative QCD and usually relies heavily on some input from experiments or simulations. It should be noted that in the Sakai-Sugimoto model we can only make reliable predictions about scattering processes with low momentum transfers (in this paper $q^2 \leq 5(\mathrm{Ge}V)^2$). This regime is very far from the Bjorken limit of DIS ($q^2 \to \infty$) which is well described by perturbative QCD. Moreover, it is beyond the scope of this paper to study the full inclusive DIS process with arbitrary final states. Therefore we limit ourselves to the contribution coming from single final state baryons with the same spin and isospin as the proton but different masses.

Electromagnetic form factors have been obtained previously using bottom-up (phenomenological) models and top-down string models. The meson form factors have been calculated in [12,13,14] (bottom-up) and [15,16,17,18] (top-down). Baryon form factors have been obtained in [19,20] (bottom-up) and [21,22,23,24] (top-down). Deep Inelastic Scattering in AdS/QCD was first investigated by Polchinski and Strassler in a bottom-up model for the case of scalar glueballs and baryon-like fermions [25]. Further development of DIS in bottom-up and top-down models include the large x regime [26,27,28,29] as well as the small x regime where Pomeron exchange dominates [30,31,32,33,34,35]. DIS structure functions have also been calculated for strongly coupled plasmas [36,37,38,39].

The discussion in section 2 is independent of a specific model realization and contains novel results that apply very generally to non-elastic electromagnetic scattering for baryons, namely a detailed derivation of electromagnetic current matrix elements, generalized form factors, structure functions and helicity amplitudes. In section 3 we briefly review the Sakai-Sugimoto model and the description of holographic baryons. Section 4 contains the derivation of the generalized Dirac and Pauli form factors in the Sakai-Sugimoto model. Numerical results for the wave functions, masses, couplings, form factors and helicity amplitudes are presented for the lowest excited states of spin 1/2 and positive parity. In section 5 we present our numerical estimate on the proton structure functions and the associated Callan-Gross relation.

2. Baryon resonances in proton electromagnetic scattering

Massless QCD with N_f flavours enjoys chiral symmetry ($U(N_f)_L \times U(N_f)_R$) at high energies. At low energies chiral symmetry is spontaneously broken and the residual symmetry is vectorial corresponding to the group $U(N_f)_V$. In the case of two flavours, the vectorial current can be written as $J_V^{\mu,a}$ where $a = (0, 1, 2, 3)$. The electromagnetic current $\mathcal{J}^\mu$ can be obtained as a combination of the isoscalar current $J_V^{\mu,0}$ and the isovector current $J_V^{\mu,3}$:

$$\mathcal{J}^\mu = \frac{1}{N_c} J_V^{\mu,0} + J_V^{\mu,3} \equiv \sum_{a=0}^{3} c_a J_V^{\mu,a} \,. \tag{2.1}$$

In this paper we are interested in the production of baryon resonances with positive parity in electromagnetic scattering of a proton. Namely, we study the transition of a proton (denoted by B) with spin 1/2, isospin 1/2 and momentum p to a baryonic resonance (denoted by B_X) with the same spin and isospin as the proton but momentum p_X and different mass. This transition is characterized by evaluating the matrix elements of electromagnetic currents between the baryonic initial and final states. We denote the spin and isospin projections of the initial (final) baryon state as s (s_X) and I_3 (I_3^X) respectively.

2.1. The generalized Dirac and Pauli form factors

Evaluating the isoscalar and isovector current operators in the baryonic states we obtain current matrix elements that can be decomposed as

$$\begin{aligned}\langle p_X, B_X, s_X, I_3^X | J_V^{\mu,a}(0) | p, B, s, I_3\rangle &= \frac{i}{2(2\pi)^3}(\tau^a)_{I_3^X I_3}\bar{u}(p_X, s_X)\Big[\gamma^\mu F^{D,a}_{BB_X}(q^2) \\ &+ \kappa_B \sigma^{\mu\nu} q_\nu F^{P,a}_{BB_X}(q^2) + i q^\mu F^{Q,a}_{BB_X}(q^2)\Big] u(p,s)\,, \quad (2.2)\end{aligned}$$

where

$$\begin{aligned} q^\mu &= (p_X - p)^\mu \quad , \quad \kappa_B = \frac{1}{m_B + m_{B_X}}\,, \\ (\tau^0)_{I_3^X I_3} &= \delta_{I_3^X I_3} \quad , \quad (\tau^a)_{I_3^X I_3} = (\sigma^a)_{I_3^X I_3} \quad a = (1,2,3)\,, \quad (2.3)\end{aligned}$$

and σ^a are the Pauli matrices. In (2.2) we have used a generalization of the Gordon decomposition identity

$$\bar{u}_{B_X}(p')\gamma^\mu u_B(p) = \bar{u}_{B_X}(p')\left[-\frac{p'^\mu + p^\mu}{m_B + m_{B_X}} + \frac{i\sigma^{\mu\nu}(p'_\nu - p_\nu)}{m_B + m_{B_X}}\right] u_B(p)\,. \quad (2.4)$$

The scalars $F^{D,a}_{BB_X}(q^2)$,$F^{P,a}_{BB_X}(q^2)$ in (2.2) are the Dirac and Pauli form factors while $F^{Q,a}_{BB_X}(q^2)$ is required by current conservation

$$\begin{aligned} 0 &= q_\mu \langle J_V^{\mu,a}(0)\rangle \sim \bar{u}(p_X, s_X)\Big[(p_X - p)_\mu \gamma^\mu F^{D,a}_{BB_X}(q^2) + i q^2 F^{Q,a}_{BB_X}(q^2)\Big] u(p,s) \\ &= \bar{u}(p_X, s_X)\Big[i(m_{B_X} - m_B) F^{D,a}_{BB_X}(q^2) + i q^2 F^{Q,a}_{BB_X}(q^2)\Big] u(p,s)\,, \quad (2.5)\end{aligned}$$

so that

$$F^{Q,a}_{BB_X}(q^2) = -\frac{1}{q^2}(m_{B_X} - m_B) F^{D,a}_{BB_X}(q^2)\,. \quad (2.6)$$

The current matrix element now takes a transverse form

$$\begin{aligned}\langle p_X, B_X, s_X | J_V^{\mu,a}(0) | p, B, s\rangle &= \frac{i}{2(2\pi)^3}(\tau^a)_{I_3^X I_3}\left(\eta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\bar{u}(p_X, s_X)\Big[\gamma_\nu F^{D,a}_{BB_X}(q^2) \\ &+ \kappa_B \sigma_{\nu\lambda} q^\lambda F^{P,a}_{BB_X}(q^2)\Big] u(p,s)\,. \quad (2.7)\end{aligned}$$

We have chosen the signature $\eta_{\mu\nu} = \mathrm{diag}(-,+,+,+)$. The relativistic Dirac spinors that represent the initial and final states can be written as

$$u(p,s) = \frac{1}{\sqrt{2E}}\begin{pmatrix} f\,\chi_s(\vec{p}) \\ \frac{\vec{p}\cdot\vec{\sigma}}{f}\,\chi_s(\vec{p})\end{pmatrix} \quad , \quad u(p_X, s_X) = \frac{1}{\sqrt{2E_X}}\begin{pmatrix} f_X\,\chi_{s_X}(\vec{p}_X) \\ \frac{\vec{p}_X\cdot\vec{\sigma}}{f_X}\,\chi_{s_X}(\vec{p}_X)\end{pmatrix}, \quad (2.8)$$

where $f = \sqrt{E + m_B}$ and $f_X = \sqrt{E_X + m_{B_X}}$. The two-component spinors $\chi_s(\vec{p})$ and $\chi_{s_X}(\vec{p}_X)$ are defined as eigenstates of the helicity operators

$$\frac{\vec{p}\cdot\vec{\sigma}}{|\vec{p}|}\chi_s(\vec{p}) = s\,\chi_s(\vec{p}) \quad , \quad \frac{\vec{p}_X\cdot\vec{\sigma}}{|\vec{p}_X|}\chi_{s_X}(\vec{p}_X) = s_X\,\chi_{s_X}(\vec{p}_X)\,. \quad (2.9)$$

The convention for gamma matrices is

$$\gamma^0 = -i \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad , \quad \gamma^i = -i \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} \quad , \quad \sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu] \, . \tag{2.10}$$

The baryon states are normalized according to their charges as

$$\begin{aligned} \langle p_X, B_X, s_X, I_3^X | Q_V^a | p, B, s, I_3 \rangle &= \langle p_X, B_X, s_X, I_3^X | J_V^{0,a}(0) | p, B, s, I_3 \rangle (2\pi)^3 \delta^3(\vec{q}) \\ &= \frac{1}{2} (\tau^a)_{I_3^X I_3} \delta^3(\vec{q}) \delta_{s_X s} \delta_{B B_X} F_{BB}^{D,a}(0) \, . \end{aligned} \tag{2.11}$$

In the case where the initial state is a proton and the final state has the same isospin polarization ($I_3^X = I_3 = 1/2$) the isoscalar and isovector charges are $N_c/2$ and $1/2$ respectively. The elastic Dirac form factors are fixed at $q^2 = 0$ as

$$F_{BB}^{D,0}(0) = N_c \quad , \quad F_{BB}^{D,3}(0) = 1 \, , \tag{2.12}$$

so we find from (2.11)

$$\langle p_X, B_X, s_X, 1/2 | p, B, s, 1/2 \rangle = \delta^3(\vec{p}_X - \vec{p}) \delta_{s_X s} \delta_{B_X B} \, . \tag{2.13}$$

The following spinor relations are very useful

$$\begin{aligned} \bar{u}(p_X, s_X) \gamma^0 u(p, s) &= -\frac{i}{2\sqrt{E E_X}} \chi_{s_X}^\dagger(\vec{p}_X) \Big[f f_X + \frac{1}{f f_X} \vec{p}_X \cdot \vec{\sigma} \vec{p} \cdot \vec{\sigma} \Big] \chi_s(\vec{p}) \\ &= -\frac{i}{2\sqrt{E E_X}} \left(\frac{f}{f_X} \right) \left[E_X + m_{B_X} + \frac{s_X s |\vec{p}_X| |\vec{p}|}{E + m_B} \right] \chi_{s_X}^\dagger(\vec{p}_X) \chi_s(\vec{p}) \, , \end{aligned} \tag{2.14}$$

$$\begin{aligned} \bar{u}(p_X, s_X) \gamma^i u(p, s) &= -\frac{i}{2\sqrt{E E_X}} \chi_{s_X}^\dagger(\vec{p}_X) \Big\{ \frac{f_X}{f} \sigma^i \vec{p} \cdot \vec{\sigma} + \frac{f}{f_X} \vec{p}_X \cdot \vec{\sigma} \sigma^i \Big\} \chi_s(\vec{p}) \\ &= -\frac{i}{2\sqrt{E E_X}} \left(\frac{f}{f_X} \right) \left[\frac{E_X + m_{B_X}}{E + m_B} s |\vec{p}| + s_X |\vec{p}_X| \right] \chi_{s_X}^\dagger(\vec{p}_X) \sigma^i \chi_s(\vec{p}) \, , \end{aligned} \tag{2.15}$$

$$\begin{aligned} \bar{u}(p_X, s_X) \sigma^{0i} q_i u(p, s) &= -\frac{i}{2\sqrt{E E_X}} \left[\frac{f_X}{f} q^i p^j - \frac{f}{f_X} (p + q)^i q^j \right] \chi_{s_X}^\dagger(\vec{p}_X) \sigma_i \sigma_j \chi_s(\vec{p}) \\ &= -\frac{i}{2\sqrt{E E_X}} \left(\frac{f}{f_X} \right) \left[\frac{E_X + m_{B_X}}{E + m_B} s |\vec{p}| - s_X |\vec{p}_X| \right] q_i \, \chi_{s_X}^\dagger(\vec{p}_X) \sigma^i \chi_s(\vec{p}) \, , \end{aligned} \tag{2.16}$$

$$\begin{aligned} \bar{u}(p_X, s_X) \sigma^{ij} q_j u(p, s) &= -\frac{1}{2\sqrt{E E_X}} \epsilon^{ijk} q_j \chi_{s_X}^\dagger(\vec{p}_X) \Big[f_X f \sigma_k - \frac{1}{f_X f} (p + q)^a p^b \sigma_a \sigma_k \sigma_b \Big] \chi_s(\vec{p}) \\ &= -\frac{1}{2\sqrt{E E_X}} \left(\frac{f}{f_X} \right) \epsilon^{ijk} q_j \left[E_X + m_{B_X} - \frac{s_X s |\vec{p}_X| |\vec{p}|}{E + m_B} \right] \chi_{s_X}^\dagger(\vec{p}_X) \sigma_k \chi_s(\vec{p}) \, . \end{aligned} \tag{2.17}$$

The Breit frame. The Breit frame is characterized by the condition $E_X = E$ which means that the photon has zero energy ($q_0 = 0$). As shown in the appendix, choosing the photon in the z axis, the the photon and baryon momenta in the Breit frame take the form

$$\begin{aligned} q^\mu &= (0,0,0,q) \quad , \quad p^\mu = (E,0,0,p_3) \quad , \quad p_X^\mu = p^\mu + q^\mu \, , \\ p_3 &= -\frac{q}{2x} \quad , \quad E = \sqrt{m_B^2 + p_3^2} \, . \end{aligned} \tag{2.18}$$

Using eq. (2.7) and the spinor relations (2.14) - (2.17) we can work out the components of the current matrix elements. In the Breit frame the current matrix elements simplify to

$$\begin{aligned} \langle p_X, B_X, s_X, I_3^X | J_V^{0,a}(0) | p, B, s, I_3 \rangle &= \frac{1}{2(2\pi)^3} (\tau^a)_{I_3^X I_3} \, \chi^\dagger_{s_X}(\vec{p}_X) \chi_s(\vec{p}) \\ &\times \left[\alpha F^{D,a}_{BB_X}(q^2) - \beta \, q^2 \kappa_B F^{P,a}_{BB_X}(q^2) \right] , \end{aligned} \tag{2.19}$$

$$\begin{aligned} \langle p_X, B_X, s_X, I_3^X | J_V^{i,a}(0) | p, B, s, I_3 \rangle &= -\frac{i}{2(2\pi)^3} (\tau^a)_{I_3^X I_3} \epsilon^{ijk} q_j \, \chi^\dagger_{s_X}(\vec{p}_X) \sigma_k \chi_s(\vec{p}) \\ &\times \left[\beta F^{D,a}_{BB_X}(q^2) + \alpha \kappa_B F^{P,a}_{BB_X}(q^2) \right] , \end{aligned} \tag{2.20}$$

where

$$\begin{aligned} \alpha &= \left(\frac{1}{2E} \right) \left(\frac{\sqrt{E + m_B}}{\sqrt{E + m_{B_X}}} \right) \left[E + m_{B_X} + (E - m_B)(1 - 2x) \right] , \\ \beta &= \left(\frac{1}{2E} \right) \left(\frac{\sqrt{E + m_B}}{\sqrt{E + m_{B_X}}} \right) \left(\frac{1}{2x} \right) \left[\frac{E + m_{B_X}}{E + m_B} + 2x - 1 \right] , \\ E &= \sqrt{m_B^2 + \frac{q^2}{4x^2}} \, . \end{aligned} \tag{2.21}$$

To get (2.19) and (2.20), the following identity proved useful,

$$s \chi^\dagger_{s_X}(\vec{p}_X) \sigma^i \chi_s(\vec{p}) = -\delta^{i3} \chi^\dagger_{s_X}(\vec{p}_X) \chi_s(\vec{p}) - i \epsilon^{i3k} \chi^\dagger_{s_X}(\vec{p}_X) \sigma^k \chi_s(\vec{p}) \, , \tag{2.22}$$

which is valid in the Breit frame only.
Note that in the elastic case $m_{B_X} = m_B$, $f_X = f$ and $\kappa_B = 1/(2m_B)$ so we obtain

$$\begin{aligned} \langle p_X, B_X, s_X, I_3^X | J_V^{0,a}(0) | p, B, s, I_3 \rangle &= \frac{1}{2(2\pi)^3} (\tau^a)_{I_3^X I_3} \left(\frac{m_B}{E} \right) \chi^\dagger_{s_X}(\vec{p}_X) \chi_s(\vec{p}) G_B^{E,a}(q^2) \, , \\ \langle p_X, B_X, s_X, I_3^X | J_V^{i,a}(0) | p, B, s, I_3 \rangle &= -\frac{1}{2(2\pi)^3} (\tau^a)_{I_3^X I_3} \left(\frac{i}{2E} \right) \epsilon^{ijk} q_j \, \chi^\dagger_{s_X}(\vec{p}_X) \sigma_k \chi_s(\vec{p}) G_B^{M,a}(q^2) \, , \end{aligned} \tag{2.23}$$

where

$$G_B^{E,a}(q^2) = F_B^{D,a}(q^2) - \frac{q^2}{4m_B^2} F_B^{P,a}(q^2) \quad , \quad G_B^{M,a}(q^2) = F_B^{D,a}(q^2) + F_B^{P,a}(q^2) \, , \tag{2.24}$$

are the elastic electric and magnetic form factors also known as the Sachs form factors.

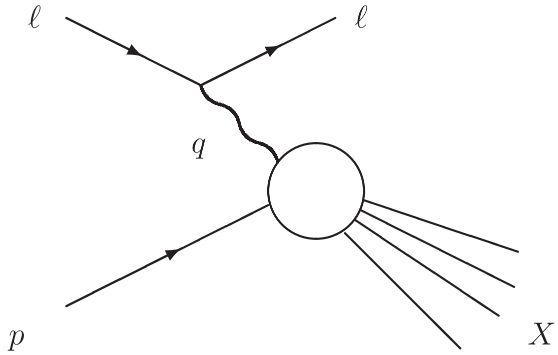

Figure 1: Exemplary diagram for a deep inelastic scattering process. A lepton ℓ exchanges a virtual photon with a hadron of momentum p.

2.2. Deep Inelastic Scattering and the proton structure functions

Deep inelastic scattering (DIS) is a primary tool to investigate the internal structure of hadrons. DIS refers to the scattering process of a lepton on a hadron. The lepton interacts with a hadron of momentum p^μ via a virtual photon of momentum q^μ (cf. figure 1). The final hadronic state is denoted by X and momentum p_X^μ. Such a process is commonly parametrized by two dynamical variables, namely the Bjorken parameter $x = -\frac{q^2}{2p\cdot q}$ and the photon virtuality q^2 (for a review of DIS, see e.g., [40]). The standard limit in DIS corresponds to the Bjorken limit of large q^2 and fixed x. In this paper we are interested in the regime of small q^2 where non-perturbative contributions are relevant.

The DIS differential cross section is determined by the hadronic tensor,

$$W^{\mu\nu} = \frac{1}{8\pi}\sum_s \int d^4x\, e^{iq\cdot x}\langle p,s|\Big[\mathcal{J}^\mu(x),\mathcal{J}^\nu(0)\Big]|p,s\rangle, \tag{2.25}$$

where $\mathcal{J}^\mu(x)$ is the electromagnetic current. Inserting the sum of the final states X we can write the hadronic tensor as

$$W^{\mu\nu} = \frac{1}{8\pi}\sum_s\sum_X (2\pi)^4\delta^4(p+q-p_X)\langle p,s|\mathcal{J}^\mu(0)|X\rangle\langle X|\mathcal{J}^\nu(0)|p,s\rangle\,. \tag{2.26}$$

The structure functions $F_1(x,q^2)$ and $F_2(x,q^2)$ are Lorentz invariant scalars that appear in the decomposition of the hadronic tensor :

$$W^{\mu\nu} = F_1(x,q^2)\Big(\eta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\Big) + \frac{2x}{q^2}F_2(x,q^2)\Big(p^\mu + \frac{q^\mu}{2x}\Big)\Big(p^\nu + \frac{q^\nu}{2x}\Big). \tag{2.27}$$

2.2.1. The contribution from resonance production

First of all we need to transform the spinors and baryon states of the previous section as

$$u(p,s) \to \frac{1}{\sqrt{2E}}\,u(p,s) \quad , \quad |p,B,s\rangle \to \frac{1}{\sqrt{2E}(2\pi)^{3/2}}|p,B,s\rangle\,, \tag{2.28}$$

in order to get the standard relativistic normalizations

$$\bar{u}(p,s)u(p,s) = 2m_B \quad , \quad \langle p_X, B_X, S_X|p, B, s\rangle = 2\sqrt{EE_X}(2\pi)^3\delta^3(\vec{p}_X - \vec{p})\delta_{s_X s} \,. \qquad (2.29)$$

Using (2.1), (2.7) and (2.28) we obtain for $I_3 = I_3^X = 1/2$,

$$\begin{aligned}\langle p_X, B_X, s_X|\mathcal{J}^\mu(0)|p, B, s\rangle \;&=\; i\left(\eta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\bar{u}(p_X, s_X)\Big[\gamma_\nu F^D_{BB_X}(q^2) \\ &\quad + \kappa_B\sigma_{\nu\lambda}q^\lambda F^P_{BB_X}(q^2)\Big]u(p,s)\,, \end{aligned}\qquad (2.30)$$

where

$$F^D_{BB_X}(q^2) = \frac{1}{2}\sum_a c_a F^{D,a}_{BB_X}(q^2) \quad , \quad F^P_{BB_X}(q^2) = \frac{1}{2}\sum_a c_a F^{P,a}_{BB_X}(q^2)\,, \qquad (2.31)$$

are the Dirac and Pauli electromagnetic form factors.
The baryonic tensor for a spin 1/2 baryon in the case where one particle is produced in the final state can be written as

$$\begin{aligned} W^{\mu\nu} \;&=\; \frac{1}{8\pi}\sum_{s,s_X}\sum_{m_{B_X}}\int\frac{d^4p_X}{(2\pi)^3}\theta(p^0_X)\delta(p^2_X + m^2_{B_X}) \\ &\quad \times(2\pi)^4\delta^4(p+q-p_X)\langle p, B, s|\mathcal{J}^\mu(0)|p_X, B_X, s_X\rangle\langle p_X, B_X, s_X|\mathcal{J}^\nu(0)|p, B\rangle \\ &=\; \frac{1}{4}\sum_{s,s_X}\sum_{m_{B_X}}\delta\left[(p+q)^2 + m^2_{B_X}\right]\langle p, B, s|\mathcal{J}^\mu(0)|p_X, B_X, s_X\rangle\langle p_X, B_X, s_X|\mathcal{J}^\nu(0)|p, B, s\rangle\,. \end{aligned}\qquad (2.32)$$

Substituting (2.30) into (2.32) we obtain

$$\begin{aligned} W^{\mu\nu} \;&=\; -\frac{1}{4}\sum_{m_{B_X}}\delta\left[(p+q)^2 + m^2_{B_X}\right]\left(\eta^{\mu\rho} - \frac{q^\mu q^\rho}{q^2}\right)\left(\eta^{\nu\sigma} - \frac{q^\nu q^\sigma}{q^2}\right) \\ &\quad \times\Big[F^D_{BB_X}(q^2)F^D_{BB_X}(q^2)\mathcal{A}_{\rho\sigma} + F^P_{BB_X}(q^2)F^P_{BB_X}(q^2)\mathcal{B}_{\rho\sigma} \\ &\quad + F^P_{BB_X}(q^2)F^D_{BB_X}(q^2)\mathcal{C}_{\rho\sigma} + F^D_{BB_X}(q^2)F^P_{BB_X}(q^2)\mathcal{D}_{\rho\sigma}\Big]\,, \end{aligned}\qquad (2.33)$$

where

$$\begin{aligned} \mathcal{A}_{\rho\sigma} \;&=\; -p^\tau(p+q)^{\bar{\tau}}\,\mathrm{tr}(\gamma_\tau\gamma_\rho\gamma_{\bar{\tau}}\gamma_\sigma) + m_B m_{B_X}\,\mathrm{tr}(\gamma_\rho\gamma_\sigma) \\ &=\; 4\Big\{\,[m_B m_{B_X} + p\cdot(p+q)]\,\eta_{\rho\sigma} - 2p_\rho p_\sigma - p_\rho q_\sigma - p_\sigma q_\rho\Big\}\,, \\ &=\; 4\Big\{\left[m_B m_{B_X} + p^2 - \frac{q^2}{2x}\right]\eta_{\rho\sigma} - 2p_\rho p_\sigma - p_\rho q_\sigma - p_\sigma q_\rho\Big\}\,, \end{aligned}\qquad (2.34)$$

$$\mathcal{B}_{\rho\sigma} \;=\; -\kappa^2_B q^\lambda q^{\bar{\lambda}}\left[m_B m_{B_X}\,\mathrm{tr}(\sigma_{\lambda\rho}\sigma_{\bar{\lambda}\sigma}) - p^\tau(p+q)^{\bar{\tau}}\,\mathrm{tr}(\sigma_{\lambda\rho}\gamma_{\bar{\tau}}\sigma_{\bar{\lambda}\sigma}\gamma_\tau)\right]$$

$$
\begin{aligned}
&= 4\kappa_B^2 q^2 \Big\{ \left[-m_B m_{B_X} + p^2 + \frac{q^2}{2x}\left(1-\frac{1}{x}\right)\right]\eta_{\rho\sigma} - 2p_\rho p_\sigma \\
&- \frac{1}{x}(q_\rho p_\sigma + q_\sigma p_\rho) + \left[m_B m_{B_X} - p^2 - \frac{q^2}{2x}\right]\frac{q_\rho q_\sigma}{q^2}\Big\}\,, \qquad (2.35)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{C}_{\rho\sigma} &= -i\kappa_B q^\lambda \left[m_B (p+q)^\tau \operatorname{tr}(\sigma_{\lambda\rho}\gamma_\tau\gamma_\sigma) + m_{B_X} p^\tau \operatorname{tr}(\sigma_{\lambda\rho}\gamma_\sigma\gamma_\tau)\right] \\
&= 4\kappa_B \Big\{ \left[-m_B(p+q)\cdot q + m_{B_X} p\cdot q\right]\eta_{\rho\sigma} + \left[m_B(p+q)_\rho - m_{B_X}p_\sigma\right] q_\sigma \Big\} \\
&= 4\kappa_B q^2 \Big\{ -\left[m_B + \frac{1}{2x}(m_{B_X} - m_B)\right]\eta_{\rho\sigma} + \left[m_B q_\rho + (m_B - m_{B_X})p_\rho\right]\frac{q_\sigma}{q^2}\Big\}, \qquad (2.36)
\end{aligned}
$$

and

$$
\begin{aligned}
\mathcal{D}_{\rho\sigma} &= i\kappa_B q^\lambda \left[m_B (p+q)^\tau \operatorname{tr}(\sigma_{\lambda\sigma}\gamma_\rho\gamma_\tau) + m_{B_X} p^\tau \operatorname{tr}(\sigma_{\lambda\sigma}\gamma_\tau\gamma_\rho)\right] \\
&= \mathcal{C}_{\sigma\rho}\,, \qquad (2.37)
\end{aligned}
$$

and we used the sum over spin formula

$$
\sum_s u(p,s)\bar{u}(p,s) = -i\gamma^\mu p_\mu + m_B\,, \qquad (2.38)
$$

and the gamma trace identities

$$
\begin{aligned}
\operatorname{tr}(\gamma_\mu\gamma_\nu) &= +4\eta_{\mu\nu}\,, \\
\operatorname{tr}(\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma) &= +4(\eta_{\mu\nu}\eta_{\rho\sigma} - \eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho}) \equiv 4\tilde{\eta}_{\mu\nu\rho\sigma}\,, \\
\operatorname{tr}(\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_\lambda\gamma_\tau) &= +4\Big[\eta_{\mu\nu}\tilde{\eta}_{\rho\sigma\lambda\tau} - \eta_{\mu\rho}\tilde{\eta}_{\nu\sigma\lambda\tau} + \eta_{\mu\sigma}\tilde{\eta}_{\nu\rho\lambda\tau} - \eta_{\mu\lambda}\tilde{\eta}_{\nu\rho\sigma\tau} + \eta_{\mu\tau}\tilde{\eta}_{\nu\rho\sigma\lambda}\Big]\,, \\
\operatorname{tr}(\sigma_{\mu\nu}\gamma_\rho\gamma_\sigma) &= -\operatorname{tr}(\sigma_{\mu\nu}\gamma_\rho\gamma_\sigma) = 4i(-\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho})\,, \\
\operatorname{tr}(\sigma_{\mu\nu}\sigma_{\rho\sigma}) &= -4(-\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho})\,, \\
\operatorname{tr}(\sigma_{\mu\nu}\gamma_\rho\sigma_{\sigma\lambda}\sigma_\tau) &= -4\Big[-\eta_{\mu\rho}(\eta_{\nu\sigma}\eta_{\lambda\tau} - \eta_{\nu\lambda}\eta_{\sigma\tau}) + \eta_{\mu\sigma}\tilde{\eta}_{\nu\rho\lambda\tau} - \eta_{\mu\lambda}\tilde{\eta}_{\nu\rho\sigma\tau} \\
&\quad + \eta_{\mu\tau}(-\eta_{\nu\sigma}\eta_{\rho\lambda} + \eta_{\nu\lambda}\eta_{\rho\sigma})\Big]\,. \qquad (2.39)
\end{aligned}
$$

Note that the terms with q_ρ or q_σ will vanish when contracting with the transverse tensors. Using (2.34),(2.35),(2.36),(2.37) in (2.33) we obtain the baryonic tensor

$$
W^{\mu\nu} = \left(\eta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) F_1(q^2,x) + \left(p^\mu + \frac{q^\mu}{2x}\right)\left(p^\nu + \frac{q^\nu}{2x}\right)\frac{2x}{q^2} F_2(q^2,x)\,, \qquad (2.40)
$$

in terms of the structure functions

$$
\begin{aligned}
F_1(q^2,x) &= \sum_{m_{B_X}} \delta\left[(p+q)^2 + m_{B_X}^2\right]\Big\{\left[-m_B(m_{B_X} - m_B) + \frac{q^2}{2x}\right](F^D_{BB_X}(q^2,x))^2 \\
&+ \left[m_B(m_{B_X} + m_B) + \frac{q^2}{2x}\left(\frac{1}{x} - 1\right)\right]\kappa_B^2\, q^2 (F^P_{BB_X}(q^2))^2 \\
&+ 2\left[m_B + \frac{1}{2x}(m_{B_X} - m_B)\right]\kappa_B\, q^2 F^P_{BB_X}(q^2) F^D_{BB_X}(q^2)\Big\}\,, \qquad (2.41)
\end{aligned}
$$

and

$$F_2(q^2,x) = \left(\frac{q^2}{x}\right)\sum_{m_{B_X}} \delta\left[(p+q)^2+m_{B_X}^2\right] \times\left[(F^D_{BB_X}(q^2))^2+\kappa_B^2 q^2 (F^P_{BB_X}(q^2))^2\right]. \tag{2.42}$$

Interestingly, we can rewrite $F_1(q^2,x)$ as a binomial squared, i.e.,

$$F_1(q^2,x) = \sum_{m_{B_X}} \delta\left[(p+q)^2+m_{B_X}^2\right]\zeta^2\left[F^D_{BB_X}(q^2,x)+F^P_{BB_X}(q^2,x)\right]^2, \tag{2.43}$$

where

$$\zeta = q\,(m_B+m_{B_X})^{-1/2}\left[m_B+\frac{1}{2x}(m_{B_X}-m_B)\right]^{1/2}. \tag{2.44}$$

Note that in the elastic case $\zeta = q/\sqrt{2}$ and $\kappa_B = 1/(2m_B)$ so that the structure functions reduce to

$$\begin{aligned} F_1(q^2,x) &= \left(\frac{q^2}{2}\right)\delta\left(q^2+2p\cdot q\right)\left[F^D_{BB}(q^2)+F^P_{BB}(q^2)\right]^2, \\ F_2(q^2,x) &= q^2\delta\left(q^2+2p\cdot q\right)\left[(F^D_{BB}(q^2))^2+\frac{q^2}{4m_B^2}(F^P_{BB}(q^2))^2\right]. \end{aligned} \tag{2.45}$$

2.2.2. *The helicity amplitudes*

It is interesting to compare the result that we have obtained for the structure functions with the standard result in terms of the helicity amplitudes [41]

$$\begin{aligned} F_1(q^2,x) &= \sum_{m_{B_X}} \delta\left[(p+q)^2+m_{B_X}^2\right] m_B^2 (G^+_{BB_X}(q^2))^2 \\ F_2(q^2,x) &= \sum_{m_{B_X}} \delta\left[(p+q)^2+m_{B_X}^2\right]\left(\frac{q^2}{2x}\right)\left(1+\frac{q^2}{4m_B^2x^2}\right)^{-1} \\ &\times \left[(G^+_{BB_X}(q^2))^2+2(G^0_{BB_X}(q^2))^2\right]. \end{aligned} \tag{2.46}$$

The helicity amplitudes $G^+_{BB_X}(q^2)$ and $G^0_{BB_X}(q^2)$ describe transitions when the initial state is a proton and the final state is a baryon of the same spin but different helicities. The amplitude G_+ (G_0) corresponds to a final baryon with helicity 1/2 ($-1/2$) and spin polarization $-1/2$ ($-1/2$).

Comparing (2.46) with our results for the structure functions we get the interesting relations

$$\begin{aligned} (G^+_{BB_X}(q^2))^2 &= \left[\frac{\zeta}{m_B}(F^D_{BB}(q^2)+F^P_{BB}(q^2))\right]^2, \\ (G^0_{BB_X}(q^2))^2 &= \left(1+\frac{q^2}{4m_B^2x^2}-\frac{\zeta^2}{2m_B^2}\right)(F^D_{BB}(q^2))^2+\left(1+\frac{q^2}{4m_B^2x^2}-\frac{\zeta^2}{2m_B^2\kappa_B^2q^2}\right)\kappa_B^2q^2(F^P_{BB}(q^2))^2 \\ &- \frac{\zeta^2}{m_B^2}F^D_{BB}(q^2)F^P_{BB}(q^2). \end{aligned} \tag{2.47}$$

3. Holographic baryons in the Sakai-Sugimoto model

As pointed out in the introduction, the Sakai-Sugimoto model provides new insight into the problem of hadronic scattering in the non-perturbative regime. Moreover, baryons have been successfully incorporated into this model by several groups [42,21,43,22]. This development was inspired by the Skyrme model [44] and Witten's original proposal of baryon vertices [45]. In this section we briefly review the Sakai-Sugimoto model and describe the construction of holographic baryons.

3.1. Review of the model

The Sakai-Sugimoto model is based on a configuration of N_c D4 branes and N_f D8-$\overline{\text{D8}}$ branes in the limit of large N_c with fixed N_f. This limit allows a supergravity description and can be interpreted as the quenching limit in QCD. The Sakai-Sugimoto model is the first string model that realizes confinement and chiral symmetry breaking. Below we describe this model in some detail.

Consider a set of N_c coincident D4-branes with a compact spatial direction in type IIA supergravity [46]. The D4-branes generate a background with the following metric, dilaton and four-form:

$$\begin{aligned} ds^2 &= \frac{u^{3/2}}{R^{3/2}}\left[\eta_{\mu\nu}dx^\mu dx^\nu + f(u)d\tau^2\right] + \frac{R^{3/2}}{u^{3/2}}\frac{du^2}{f(u)} + R^{3/2}u^{1/2}d\Omega_4^2\,, \\ f(u) &= 1-\frac{u_*^3}{u^3} \quad , \quad e^\phi = g_s\frac{u^{3/4}}{R^{3/4}} \quad , \quad F_4 = \frac{(2\pi l_s)^3 N_c}{V_{S^4}}\epsilon_4\,, \end{aligned} \tag{3.1}$$

where u_* is the tip of the cigar geometry generated by the D4 branes and $R = (\pi g_s N_c)^{1/3}\sqrt{\alpha'}$. The τ coordinate is compact and in order to avoid conical singularities the τ period is fixed as

$$\delta\tau = \frac{4\pi}{3}\frac{R^{3/2}}{u_*^{1/2}}\,. \tag{3.2}$$

As a consequence, we get a 4-d effective mass scale

$$M_* = \frac{2\pi}{\delta\tau} = \frac{3}{2}\frac{u_*^{1/2}}{R^{3/2}}\,. \tag{3.3}$$

The τ compactification is introduced as a mechanism of supersymmetry breaking and confinement. Imposing anti-periodic conditions for the fermionic states we get at low energies a four-dimensional non-supersymmetric strongly coupled $U(N_c)$ theory at large N_c with 't Hooft constant given by

$$\lambda = g_{YM}^2 N_c = (2\pi M_*)\, g_s N_c\, l_s\,. \tag{3.4}$$

It is convenient to define a pair of dimensionless coordinates y and z defined by the relations

$$u = u_*\left(1+y^2+z^2\right)^{1/3} \equiv u_* K_{y,z}^{1/3} \quad , \quad \tau = \frac{\delta\tau}{2\pi}\arctan\left(\frac{z}{y}\right)\,. \tag{3.5}$$

In terms of these coordinates the metric takes the form

$$\begin{aligned} \mathrm{d}s^2 &= u_*^{3/2}R^{-3/2}K_{y,z}^{1/2}\,\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu \,+\, \frac{4}{9}R^{3/2}u_*^{1/2}\frac{K_{y,z}^{-5/6}}{y^2+z^2}\Big[(z^2+y^2K_{y,z}^{1/3})\mathrm{d}z^2 \\ &+ \,(y^2+z^2K_{y,z}^{1/3})\mathrm{d}y^2+2yz(1-K_{y,z}^{1/3})\mathrm{d}y\mathrm{d}z\Big] + R^{3/2}u_*^{1/2}K_{y,z}^{1/6}\mathrm{d}\Omega_4^2\,. \end{aligned} \tag{3.6}$$

Now consider N_f coincident D8-$\overline{\text{D8}}$ probe branes living in the background generated by the N_c D4-branes. The probe approximation is guaranteed by the condition $N_f \ll N_c$. The N_f D8 branes introduce quark degrees of freedom as fundamental strings extending from the D4 branes to the D8 branes. The dynamics of the D8 and $\overline{\text{D8}}$ branes is dictated by the DBI action. It turns out that the solution to the DBI equations smoothly merges the D8 and $\overline{\text{D8}}$ branes in the infrared region (small u). This is a geometrical realization of chiral symmetry breaking $U(N_f)\times U(N_f) \to U(N_f)$. In the simplest case the solution is just $y = 0$ (antipodal solution) and the induced D8-$\overline{\text{D8}}$ metric takes the form,

$$\mathrm{d}s_{D8} = u_*^{3/2}R^{-3/2}K_z^{1/2}\,\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu \,+\, \frac{4}{9}R^{3/2}u_*^{1/2}K_z^{-5/6}\mathrm{d}z^2 + R^{3/2}u_*^{1/2}K_z^{1/6}\mathrm{d}\Omega_4^2\,, \tag{3.7}$$

where $K_z = 1+z^2$. Considering small gauge field fluctuations $\mathcal{A}_\mu, \mathcal{A}_z$ depending only in x^μ and z directions the action of the D8-$\overline{\text{D8}}$ branes reduces to a five-dimensional $U(N_f)$ Yang Mills-Chern Simons (YM-CS) action in a curved background. The action of the model reads

$$\begin{aligned} &S = S_{\text{YM}} + S_{\text{CS}}\,, \\ &S_{\text{YM}} = -\kappa \int d^4x dz \;\text{tr}\;\left[\frac{1}{2}K_z^{-1/3}\eta^{\mu\rho}\eta^{\nu\sigma}\mathcal{F}_{\mu\nu}\mathcal{F}_{\rho\sigma} + M_*^2\,K_z\eta^{\mu\nu}\mathcal{F}_{\mu z}\mathcal{F}_{\nu z}\right]\,, \\ &S_{\text{CS}} = \frac{N_c}{24\pi^2}\int_{M^4\times\mathbb{R}} \text{tr}\left(\mathcal{A}\mathcal{F}^2 - \frac{i}{2}\mathcal{A}^3\mathcal{F} - \frac{1}{10}\mathcal{A}^5\right)\;. \end{aligned} \tag{3.8}$$

where $\kappa = \lambda N_c/(216\pi^3)$. Here, $\mu,\nu = 0,1,2,3$ are four-dimensional Lorentz indices, and z corresponds to the fifth dimension. The quantity $\mathcal{A} = \mathcal{A}_\alpha dx^\alpha = \mathcal{A}_\mu dx^\mu + \mathcal{A}_z dz \;\; (\alpha = 0,1,2,3,z)$ is the five-dimensional $U(N_f)$ gauge field and $\mathcal{F} = \frac{1}{2}\mathcal{F}_{\alpha\beta}dx^\alpha\wedge dx^\beta = d\mathcal{A} + i\mathcal{A}\wedge\mathcal{A}$ is its field strength.

3.2. Vector meson dominance

The gauge field $\mathcal{A}_\mu(x,z)$ can be expanded, in the $\mathcal{A}_z = 0$ gauge, as

$$\mathcal{A}_\mu(x,z) = \hat{\mathcal{V}}_\mu(x) + \hat{\mathcal{A}}_\mu(x)\psi_0(z) + \sum_{n=1}^{\infty}\left[v_\mu^n(x)\psi_{2n-1}(z) + a_\mu^n(x)\psi_{2n}(z)\right]\,, \tag{3.9}$$

where

$$\begin{aligned} \hat{\mathcal{V}}_\mu(x) &= \frac{1}{2}U^{-1}\left[\mathcal{A}_\mu^L + \partial_\mu\right]U + \frac{1}{2}U\left[\mathcal{A}_\mu^R + \partial_\mu\right]U^{-1}\,, \\ \hat{\mathcal{A}}_\mu(x) &= \frac{1}{2}U^{-1}\left[\mathcal{A}_\mu^L + \partial_\mu\right]U - \frac{1}{2}U\left[\mathcal{A}_\mu^R + \partial_\mu\right]U^{-1}\,, \end{aligned}$$

$$U(x) \;=\; e^{\frac{i\pi(x)}{f_\pi}} \quad , \quad \mathcal{A}^{L(R)}_\mu(x) = \mathcal{A}^V_\mu(x) \pm \mathcal{A}^A_\mu(x)\,, \tag{3.10}$$

and the $\psi_n(z)$ modes satisfy

$$\kappa \int dz\, K_z^{-1/3} \psi_n(z)\psi_m(z) = \delta_{nm} \quad , \quad -K_z^{1/3}\partial_z\left[K_z \partial_z \psi_n(z)\right] = \lambda_n\, \psi_n(z)\,. \tag{3.11}$$

Using the Kaluza-Klein expansion (3.9) and integrating the z coordinate we get a four-dimensional effective lagrangian of mesons and external $U(1)$ fields. The vector (axial vector) mesons are represented by the fields $v^n_\mu(x)$ ($a^n_\mu(x)$) and correspond to the modes $\psi_{2n-1}(z)$ ($\psi_{2n}(z)$). The pion is represented by the field $\pi(x)$ and corresponds to the mode $\psi_0(z)$. In addition, we have external $U(1)$ vector (axial) fields represented by $\mathcal{A}^V_\mu$ ($\mathcal{A}^A_\mu$).

In order to have a diagonal kinetic term, the vector mesons are redefined as $\tilde{v}^n_\mu = v^n_\mu + (g_{v^n}/M^2_{v^n})\mathcal{V}_\mu$ and the quadratic terms in the vector sector take the form [11] :

$$\mathcal{L}_2 \;=\; \frac{1}{2}\sum_n \left[\mathrm{Tr}\left(\partial_\mu \tilde{v}^n_\nu - \partial_\nu \tilde{v}^n_\mu\right)^2 + 2M^2_{v^n}\,\mathrm{Tr}\left(\tilde{v}^n_\mu - \frac{g_{v^n}}{M^2_{v^n}}\mathcal{V}_\mu\right)^2\right] ,$$

where

$$M^2_{v^n} = \lambda_{2n-1} M^2_* \quad , \quad g_{v^n} = \kappa\, M^2_{v^n} \int dz\, K_z^{-1/3}\psi_{2n-1}(z)\,.$$

The mixed term $g_{v^n}\tilde{v}^n_\mu \mathcal{V}^\mu$ represents the decay of the photon into vector mesons which is a holographic realization of vector meson dominance.

3.3. Holographic baryons

We restrict ourselves to the case $N_f = 2$. The $U(2)$ gauge field $\mathcal{A}$ can be decomposed as

$$\mathcal{A} = A + \widehat{A}\,\frac{\mathbf{1}_2}{2} = A^i \frac{\tau^i}{2} + \widehat{A}\,\frac{\mathbf{1}_2}{2} = \sum_{a=0}^{3} \mathcal{A}^a\,\frac{\tau^a}{2}\ , \tag{3.12}$$

where τ^i ($i = 1,2,3$) are Pauli matrices and $\tau^0 = \mathbf{1}_2$ is a unit matrix of dimension 2. Thus, the equations of motion are given by

$$\begin{aligned}
&-\kappa\left(K_z^{-1/3}\partial_\nu \widehat{F}^{\mu\nu} + \partial_z(K_z \widehat{F}^{\mu z})\right) + \frac{N_c}{128\pi^2}\epsilon^{\mu\alpha_2\ldots\alpha_5}\left(F^a_{\alpha_2\alpha_3}F^a_{\alpha_4\alpha_5} + \widehat{F}_{\alpha_2\alpha_3}\widehat{F}_{\alpha_4\alpha_5}\right) = 0,\\
&-\kappa\left(K_z^{-1/3}\nabla_\nu F^{\mu\nu} + \nabla_z(K_z F^{\mu z})\right)^a + \frac{N_c}{64\pi^2}\epsilon^{\mu\alpha_2\ldots\alpha_5}F^a_{\alpha_2\alpha_3}\widehat{F}_{\alpha_4\alpha_5} = 0,\\
&-\kappa K_z \partial_\nu \widehat{F}^{z\nu} + \frac{N_c}{128\pi^2}\epsilon^{z\mu_2\ldots\mu_5}\left(F^a_{\mu_2\mu_3}F^a_{\mu_4\mu_5} + \widehat{F}_{\mu_2\mu_3}\widehat{F}_{\mu_4\mu_5}\right) = 0,\\
&-\kappa K_z\left(\nabla_\nu F^{z\nu}\right)^a + \frac{N_c}{64\pi^2}\epsilon^{z\mu_2\ldots\mu_5}F^a_{\mu_2\mu_3}\widehat{F}_{\mu_4\mu_5} = 0,
\end{aligned}$$

where $\nabla_\alpha = \partial_\alpha + iA_\alpha$ is the covariant derivative. The baryon in this model corresponds to a soliton with a nontrivial instanton number in the four-dimensional space parameterized by x^M ($M = 1,2,3,z$). The instanton number is interpreted as the baryon number N_B, where

$$N_B = \frac{1}{64\pi^2}\int d^3x dz\, \epsilon_{M_1M_2M_3M_4}F^a_{M_1M_2}F^a_{M_3M_4}\ . \tag{3.13}$$

The equations of motion are complicated nonlinear differential equations in a curved space-time, so it is difficult to find a general analytic solution corresponding to the baryons.

3.3.1. Classical solution

Since we are working in the large λ regime, we can employ a $1/\lambda$ expansion. It can be easily observed that $S_{\rm CS}$ will be subleading compared to $S_{\rm YM}$, and therefore the leading contribution to the instanton mass comes from the YM action. As it turns out [42], it is possible to focus on a small region around the center of the instanton at $z = 0$ (because the instanton size will scale as $\lambda^{-1/2}$), where the warp factor K_z is approximately 1. The corresponding field equations will be solved by a BPST instanton with infinitesimal size $\rho \to 0$. As explained in [42], including the contributions to the field equations from the CS term will induce a non-vanishing $U(1)$ electric field $\widehat{A}_0$ and will stabilize the size of the instanton (determined by the minimum of the effective potential for ρ) at a finite value. The classical solution near $z = 0$ corresponds to a static baryon configuration and is given by

$$A^{\rm cl}_M = -\, i f(\xi) g \partial_M g^{-1} \ , \quad \widehat{A}^{\rm cl}_0 = \frac{N_c}{8\pi^2\kappa} \frac{1}{\xi^2}\left[1 - \frac{\rho^4}{(\rho^2+\xi^2)^2}\right] \ , \quad A_0 = \widehat{A}_M = 0. \tag{3.14}$$

with the definitions

$$f(\xi) = \frac{\xi^2}{\xi^2+\rho^2} \ , \quad g(x) = \frac{(z-Z) - i(\vec{x}-\vec{X})\cdot\vec{\tau}}{\xi} \ , \quad \xi = \sqrt{(z-Z)^2 + |\vec{x}-\vec{X}|^2}, \tag{3.15}$$

where $X^M = (X^1, X^2, X^3, Z) = (\vec{X}, Z)$ determines the position in the spatial $\mathbb{R}^4$ direction. The effective potential for ρ and Z can be calculated by taking into account the nontrivial z-dependence of the background (through K_z) at order λ^{-1} and reads

$$V_{\rm eff}(\rho, Z) = M_0 \left(1 + \frac{\rho^2}{6} + \frac{N_c^2}{5M_0^2}\frac{1}{\rho^2} + \frac{Z^2}{3}\right), \tag{3.16}$$

where $M_0 = 8\pi^2\kappa M_*$ is the minimal (groundstate) mass of the baryons. The effective potential is minimized at

$$\rho^2_{\rm cl} = \frac{N_c}{M_0}\sqrt{\frac{6}{5}} \ , \quad Z_{\rm cl} = 0. \tag{3.17}$$

3.3.2. Quantization

The quantization of the solitons is facilitated by employing the moduli space approximation method to study a quantum mechanical problem on the instanton moduli space. For a more detailled discussion, the interested reader is referred to refs. [42,22]. Here we merely present the results for the wavefunctions and energies of the lowest excited baryon states in the slowly moving (pseudo-) moduli approximation. These (pseudo-) moduli are:

$$X^i(t) \ , \ Z(t) \ , \ \rho(t) \ , \ a^I(t) \ , \tag{3.18}$$

where X^i and Z represent the center-of-mass position of the soliton, while ρ is the size of the soliton (instanton-like in the $SU(2)$ sector) and a^I $(I = 1, 2, 3, 4)$ determines the orientation of the instanton in the $SU(2)$ group space, with the condition $(a^I)^2 = 1$. The $SU(2)$ gauge field takes the form

$$A_M = V A_M^{\rm cl} V^{-1} - iV\partial_M V^{-1}\,, \tag{3.19}$$

where $A_M^{\rm cl}$ is given by eq. (3.14) and V satisfies the Gauss law constraint

$$-\,iV^{-1}\dot{V} = -\dot{X}^M A_M^{\rm cl} + \chi^a f(\xi) g \frac{\tau^a}{2} g^{-1}\,, \tag{3.20}$$

with

$$\chi^a = -i\,{\rm tr}(\tau^a \mathbf{a}^{-1}\dot{\mathbf{a}}) \quad , \quad \mathbf{a} = a_4 + i a_a \tau^a\,. \tag{3.21}$$

Inserting (3.19) into the effective action we get the Lagrangian of collective motion of the soliton

$$L = \frac{M_0}{2}(\dot{\vec{X}}^2 + \dot{Z}^2) + M_0 \sum_{I=1}^{4} (\dot{\rho a_I})(\dot{\rho a_I}) - V_{\rm eff}(\rho, Z)\,. \tag{3.22}$$

Quantizing this system we find the baryon wavefunctions as eigenstates of the Hamiltonian. The relevant quantum numbers are $B = (l, I_3, n_\rho, n_z)$ and its spin s. For example, baryon wavefunctions with $B_n = (1, +1/2, 0, n)$ can be written as

$$|\, B_n \uparrow\rangle \propto R(\rho)\psi_{B_n}(Z)(a_1 + ia_2)\,, \tag{3.23}$$

where

$$\begin{aligned} R(\rho) &= \rho^{-1+2\sqrt{1+N_c^2/5}} e^{-\frac{M_0}{\sqrt{6}}\rho^2}\,, \\ \Psi_{B_n}(Z) &= \left(\frac{(2M_0)^{1/4}}{6^{1/8}\,\pi^{1/4}\,2^{n/2}\,\sqrt{n!}}\right) H_n(\sqrt{2M_0}6^{-1/4}Z){\rm e}^{-\frac{M_0}{\sqrt{6}}Z^2}\,. \end{aligned} \tag{3.24}$$

The baryon masses can be easily gleaned from the relevant Hamiltonians and the resulting mass formula reads,

$$M = M_0 + \sqrt{\frac{(\ell+1)^2}{6} + \frac{2}{15}N_c^2} + \frac{2(n_\rho + n_z) + 2}{\sqrt{6}} =: \widetilde{M}_0 + \frac{2n_z}{\sqrt{6}}. \tag{3.25}$$

One observation is in order: Since the photon couples to the baryons via vector mesons as a direct consequence of vector meson dominance and since the vector meson wavefunctions only depend on the coordinate z, it is clear that the intial and final state baryons must have the same ρ quantum number due to orthonormality, while they may differ in the z quantum number, due to the additional contribution from the vector mesons to the relevant coupling constants etc.

3.3.3. *Extension of the soliton solution to large z*

The classical solution (3.14) is valid only near $z = 0$. This solution can be extended to large z as long as we require $\rho \ll \xi$ which is the condition of small size for the skyrmion. Under this condition the equations of motion linearize and the solutions can be found by defining Green's functions corresponding to the curved space generated by K_z :

$$
\begin{aligned}
G(\vec{x}, z, \vec{X}, Z) &= \kappa \sum_{n=1}^{\infty} \psi_n(z)\psi_n(Z) Y_n(|\vec{x} - \vec{X}|) \\
H(\vec{x}, z, \vec{X}, Z) &= \kappa \sum_{n=0}^{\infty} \phi_n(z)\phi_n(Z) Y_n(|\vec{x} - \vec{X}|)\,,
\end{aligned}
\tag{3.26}
$$

where $\psi_n(z)$ is the complete set of vector meson eigenfunctions, and $\phi_n(z)$ is another set defined by

$$
\phi_0(z) = \frac{1}{\sqrt{\kappa\pi} K_z} \quad , \quad \phi_n(z) = \frac{1}{\sqrt{\lambda_n}} \partial_z \psi_n(z) \quad (n = 1, 2, \dots)\,,
\tag{3.27}
$$

and $Y_n(r)$ is the Yukawa potential

$$
Y_n(r) = -\frac{1}{4\pi} \frac{e^{-\sqrt{\lambda_n} r}}{r}\,.
\tag{3.28}
$$

The gauge field solutions found in [22] for the case $\rho \ll \xi$ can be written as

$$
\begin{aligned}
\hat{A}_0 &= -\frac{N_c}{2\kappa} G(\vec{x}, z, \vec{X}, Z)\,, \\
\hat{A}_i &= \frac{N_c}{2\kappa} \left\{ \dot{X}^i + \frac{\rho^2}{2} \left[\frac{\chi^a}{2} \left(\epsilon^{iaj} \frac{\partial}{\partial X^j} - \delta^{ia} \frac{\partial}{\partial Z} \right) + \frac{\dot{\rho}}{\rho} \frac{\partial}{\partial X^i} \right] \right\} G(\vec{x}, z, \vec{X}, Z)\,, \\
\hat{A}_z &= \frac{N_c}{2\kappa} \left[\dot{Z} + \frac{\rho^2}{2} \left(\frac{\chi^a}{2} \frac{\partial}{\partial X^a} + \frac{\dot{\rho}}{\rho} \frac{\partial}{\partial Z} \right) \right] H(\vec{x}, z, \vec{X}, Z)\,, \\
A_0^\Lambda &= 2\pi^2\rho^2 \Big\{ 2i\, \mathbf{a}\, \dot{\mathbf{a}}^{-1} + 2\pi^2\rho^2 \mathbf{a}\tau^a \mathbf{a}^{-1} \left[\dot{X}^i \left(\epsilon^{iaj} \frac{\partial}{\partial X^j} - \delta^{ia} \frac{\partial}{\partial Z} \right) + \dot{Z} \frac{\partial}{\partial X^a} \right] \Big\} G(\vec{x}, z, \vec{X}, Z)\,, \\
A_i^\Lambda &= -2\pi^2\rho^2 \mathbf{a}\tau^a \mathbf{a}^{-1} \left(\epsilon^{iaj} \frac{\partial}{\partial X^j} - \delta^{ia} \frac{\partial}{\partial Z} \right) G(\vec{x}, z, \vec{X}, Z)\,, \\
A_z^\Lambda &= -2\pi^2\rho^2 \mathbf{a}\tau^a \mathbf{a}^{-1} \frac{\partial}{\partial X^a} H(\vec{x}, z, \vec{X}, Z)\,,
\end{aligned}
\tag{3.29}
$$

where

$$
A_\alpha^\Lambda = \Lambda A_\alpha \Lambda^{-1} - i\Lambda \partial_\alpha \Lambda \quad , \quad \Lambda = \mathbf{a} g^{-1} V^{-1}\,.
\tag{3.30}
$$

4. Generalized baryon form factors in the Sakai-Sugimoto model

Now we are going to extract the generalized Dirac and Pauli form factors by comparing the matrix element of the vectorial current in (2.7) with the one that can be obtained from the

Sakai-Sugimoto model. The latter, denoted here by $J^{\mu,a}_{V(SS)}$, can be gleaned from holography as [22]

$$J^{\mu,a}_{V(SS)} = -\kappa\Big\{ \lim_{z\to\infty} \left[K_z \mathcal{F}^{\rm cl}_{\mu z}\right] + \lim_{z\to-\infty} \left[K_z \mathcal{F}^{\rm cl}_{\mu z}\right] \Big\}\,, \tag{4.1}$$

where $\mathcal{F}^{\rm cl}_{\mu z}$ is the field strength associated with the classical field (3.29). When comparing the vectorial current of (2.7) with the one in the Sakai-Sugimoto model we will use the following prescription

$$\eta_\mu \langle p_X, B_X, s_X | J^{\mu,a}_V(0) | p, B, s\rangle = \eta_\mu \langle p_X, B_X, s_X | J^{\mu,a}_{V(SS)}(0) | p, B, s\rangle \tag{4.2}$$

where $\eta_\mu = (\eta_0, \vec{\eta})$ is the polarization of the photon and we choose to work with transverse photons satisfying the relation $\eta_\mu q^\mu = 0$ in order to avoid the discussion of current anomalies.

4.1. Electromagnetic currents in the Sakai-Sugimoto model

Using (3.29) and (4.1) we get the holographic currents in the Sakai-Sugimoto model [22] :

$$\begin{aligned}
J^{0,0}_{V(SS)}(x) &= \frac{N_c}{2} G_V\,, \\
J^{i,0}_{V(SS)}(x) &= -\frac{N_c}{2}\Big\{ \dot{Z}\partial^i H_V - \dot{X}^i G_V - \frac{S_a}{16\pi^2\kappa}\left[(\partial^i\partial^a - \delta^{ia}\partial^2) H_V + \epsilon^{ija}\partial_j G_V\right]\Big\}\,, \\
J^{0,c}_{V(SS)}(x) &= 2\pi^2\kappa\Big\{ \rho^2\,{\rm tr}[\tau^c \partial_0(\mathbf{a}\tau^a\mathbf{a}^{-1})]\partial_a H_V + \frac{I^c}{2\pi^2\kappa} G_V \\
&- \rho^2\,{\rm tr}[\tau^c \mathbf{a}\tau^a\mathbf{a}^{-1}]\dot{X}^i\left[(\partial_a\partial_i - \delta_{ia}\partial^2) H_V + \epsilon^{ija}\partial_j G_V\right]\Big\}\,, \\
J^{i,c}_{V(SS)}(x) &= -2\pi^2\kappa\rho^2\,{\rm tr}[\tau^c\mathbf{a}\tau_a\mathbf{a}^{-1}]\left[(\partial^i\partial^a - \delta^{ia}\partial^2) H_V + \epsilon^{ija}\partial_j G_V\right]\,,
\end{aligned} \tag{4.3}$$

where

$$\begin{aligned}
G_V &= -\sum_n g_{v^n}\psi_{2n-1}(Z) Y_{2n-1}(|\vec{x} - \vec{X}|)\,, \\
H_V &= -\sum_n \frac{g_{v^n}}{\lambda_{2n-1}}\partial_Z\psi_{2n-1}(Z) Y_{2n-1}(|\vec{x} - \vec{X}|)\,, \\
\dot{Z} &= -\frac{i}{M_0}\partial_Z = \frac{P_Z}{M_0} \quad , \quad \dot{X}^i = -\frac{i}{M_0}\frac{\partial}{\partial X^i} = \frac{P^i}{M_0}\,,
\end{aligned} \tag{4.4}$$

and

$$S^a = 4\pi^2\kappa\rho^2\chi_a = -i4\pi^2\kappa\rho^2\,{\rm tr}(\tau^a\mathbf{a}^{-1}\dot{\mathbf{a}}) \quad , \quad I^a = -i4\pi^2\kappa\rho^2\,{\rm tr}(\tau^a\mathbf{a}\dot{\mathbf{a}}^{-1})\,, \tag{4.5}$$

are the spin and isospin operators. Note that

$$\dot{Z}(\partial^i H_V) - \dot{X}^i G_V = \frac{1}{M_0}\Big[(\partial^i H_V) P_Z - G_V P^i\Big]. \tag{4.6}$$

Here we used the relation $\partial_Z H_V = -G_V$.
Defining the Fourier transform as

$$\tilde{J}^{\mu,a}_{V(SS)}(\vec{k}) = \int d^3\vec{x} e^{-i\vec{k}\cdot x} J^{\mu,a}_{V(SS)}(x)\,, \tag{4.7}$$

and using the identity

$$\int d^3\vec{x} e^{-i\vec{k}\cdot x} Y_{2n-1}(|\vec{x}-\vec{X}|) = -\frac{e^{-i\vec{k}\cdot\vec{X}}}{\vec{k}^2+\lambda_{2n-1}}\,, \tag{4.8}$$

we find

$$\tilde{J}^{0,0}_{V(SS)}(\vec{k}) = \frac{N_c}{2} e^{-i\vec{k}\cdot\vec{X}} \sum_n \frac{g_{v^n}\psi_{2n-1}(Z)}{\vec{k}^2+\lambda_{2n-1}}\,, \tag{4.9}$$

$$\begin{aligned}\tilde{J}^{i,0}_{V(SS)}(\vec{k}) &= \frac{N_c}{2} e^{-i\vec{k}\cdot\vec{X}} \Big\{ \sum_n \frac{g_{v^n}\psi_{2n-1}(Z)}{\vec{k}^2+\lambda_{2n-1}} \left[\frac{P^i}{M_0} + \frac{i}{16\pi^2\kappa}\epsilon^{ija}k_j S_a\right] \\ &- \sum_n \frac{g_{v^n}\partial_Z\psi_{2n-1}(Z)}{\lambda_{2n-1}(\vec{k}^2+\lambda_{2n-1})}\left[\frac{k^i}{M_0}\partial_Z + \frac{1}{16\pi^2\kappa}(k^i k^a - \vec{k}^2\delta^{ia})S_a\right]\Big\}\,, \end{aligned}\tag{4.10}$$

$$\begin{aligned}\tilde{J}^{0,c}_{V(SS)}(\vec{k}) &= 2\pi^2\kappa\, e^{-i\vec{k}\cdot\vec{X}} \Big\{ \sum_n \frac{g_{v^n}\psi_{2n-1}(Z)}{\vec{k}^2+\lambda_{2n-1}} \left[\frac{I^c}{2\pi^2\kappa} - \frac{i}{M_0}\epsilon^{ija}P_i k_j \rho^2\,\mathrm{tr}(\tau^c \mathbf{a}\tau_a\mathbf{a}^{-1})\right] \\ &+ \sum_n \frac{g_{v^n}\partial_Z\psi_{2n-1}(Z)}{\lambda_{2n-1}(\vec{k}^2+\lambda_{2n-1})}\left[ik_i\rho^2\,\mathrm{tr}[\tau^c\partial_0(\mathbf{a}\tau^i\mathbf{a}^{-1})] + \frac{1}{M_0}(\vec{P}\cdot\vec{k}k_i - \vec{k}^2 P_i)\rho^2\,\mathrm{tr}[\tau^c\mathbf{a}\tau^i\mathbf{a}^{-1}]\right]\Big\}\,, \end{aligned}\tag{4.11}$$

$$\begin{aligned}\tilde{J}^{i,c}_{V(SS)}(\vec{k}) &= 2\pi^2\kappa\, e^{-i\vec{k}\cdot\vec{X}} \Big[-i\sum_n \frac{g_{v^n}\psi_{2n-1}(Z)}{\vec{k}^2+\lambda_{2n-1}}\epsilon^{ija}k_j \\ &+ \sum_n \frac{g_{v^n}\partial_Z\psi_{2n-1}(Z)}{\lambda_{2n-1}(\vec{k}^2+\lambda_{2n-1})}(k^i k^a - \vec{k}^2\delta^{ia})\Big]\rho^2\,\mathrm{tr}(\tau^c\mathbf{a}\tau_a\mathbf{a}^{-1})\,. \end{aligned}\tag{4.12}$$

Note that one term arising from $\dot{Z}$ cancels with another from $\dot{X}^i$ and we have used the relation $\partial_Z^2\psi_n(Z) \approx -\lambda_n\psi_n(Z)$. Now we calculate the expectation values of the Sakai-Sugimoto currents :

$$\langle p_X, B_X, s_X | J^{\mu,a}_{V(SS)}(0)|p,B,s\rangle = \int \frac{d^3\vec{k}}{(2\pi)^3}\langle p_X, B_X, s_X|\tilde{J}^{\mu,a}_{V(SS)}(\vec{k})|p,B,s\rangle\,. \tag{4.13}$$

We define the baryon states as

$$\begin{aligned} |\vec{p}, B, s, I_3\rangle &= \frac{1}{(2\pi)^{3/2}} e^{i\vec{p}\cdot\vec{X}}|n_B\rangle|n_\rho\rangle|s,I_3\rangle_R\,, \\ |\vec{p}_X, B_X, s_X, I_3^X\rangle &= \frac{1}{(2\pi)^{3/2}} e^{i\vec{p}_X\cdot\vec{X}}|n_{B_X}\rangle|n_\rho\rangle|s_X, I_3^X\rangle_R\,. \end{aligned}\tag{4.14}$$

Here we make use of the results and definitions of a recent publication [58], in which a relativistic generalization of baryon states and wavefunctions was discussed in detail. In particular, the spin and isospin part was defined as

$$|s, I_3\rangle_R = \frac{1}{\sqrt{2E}}\begin{pmatrix} f\,|s,I_3\rangle \\ \frac{s|\vec{p}|}{f}\,|s,I_3\rangle\end{pmatrix}\,,$$

$$\langle s_X, I_3^X|_R = \frac{1}{\sqrt{2E_X}}\left(f_X\,\langle s_X, I_3^X| \quad - \frac{s_X|\vec{p}_X|}{f_X}\,\langle s_X, I_3^X|\right)\,, \tag{4.15}$$

where $|s, I_3\rangle$ and $\langle s_X, I_3^X|$ are the non-relativistic initial and final states associated with the spin and isospin operators. Evaluating the currents in these states we get

$$\langle J^{0,0}_{V(SS)}(0)\rangle = \frac{1}{(2\pi)^3}\frac{N_c}{2}\langle s_X, I_3^X|s, I_3\rangle_R F^1_{BB_X}(\vec{q}^2)\,, \tag{4.16}$$

$$\begin{aligned}\langle J^{i,0}_{V(SS)}(0)\rangle &= \frac{1}{(2\pi)^3}\frac{N_c}{2}\langle s_X, I_3^X|_R\Big\{F^1_{BB_X}(\vec{q}^2)\left[\frac{p^i}{M_0} - \frac{i}{16\pi^2\kappa}\epsilon^{ija}q_j S_a\right]\\ &+ \frac{q^i}{M_0}F^3_{BB_X}(\vec{q}^2) - \frac{1}{16\pi^2\kappa}F^2_{BB_X}(\vec{q}^2)(q^iq^a - \vec{q}^2\delta^{ia})S_a\Big\}|s, I_3\rangle_R\,,\end{aligned} \tag{4.17}$$

$$\begin{aligned}\langle J^{0,c}_{V(SS)}(0)\rangle &= 2\pi^2\kappa\,\frac{1}{(2\pi)^3}\langle n_\rho|\langle s_X, I_3^X|_R\Big\{F^1_{BB_X}(\vec{q}^2)\left[\frac{I^c}{2\pi^2\kappa} + \frac{i}{M_0}\epsilon^{ija}p_iq_j\rho^2\,\mathrm{tr}(\tau^c\mathbf{a}\tau_a\mathbf{a}^{-1})\right]\\ &+ F^2_{BB_X}(\vec{q}^2)\Big[-iq_i\rho^2\,\mathrm{tr}[\tau^c\partial_0(\mathbf{a}\tau^i\mathbf{a}^{-1})]\\ &+ \frac{1}{M_0}(\vec{P}\cdot\vec{q}q_i - \vec{q}^2P_i)\rho^2\,\mathrm{tr}[\tau^c\mathbf{a}\tau^i\mathbf{a}^{-1}]\Big]\Big\}|n_\rho\rangle|s, I_3\rangle_R\,,\end{aligned} \tag{4.18}$$

$$\begin{aligned}\langle J^{i,c}_{V(SS)}(0)\rangle &= 2\pi^2\kappa\,\frac{1}{(2\pi)^3}\Big[iF^1_{BB_X}(\vec{q}^2)\epsilon^{ija}q_j + F^2_{BB_X}(\vec{q}^2)(q^iq^a - \vec{q}^2\delta^{ia})\Big]\\ &\times\ \langle n_\rho|\rho^2|n_\rho\rangle\langle s_X, I_3^X|_R\,\mathrm{tr}(\tau^c\mathbf{a}\tau_a\mathbf{a}^{-1})|s, I_3\rangle_R\,.\end{aligned} \tag{4.19}$$

where

$$\begin{aligned}F^1_{BB_X}(\vec{q}^2) &= \sum_n \frac{g_{v^n}\langle n_{B_X}|\psi_{2n-1}(Z)|n_B\rangle}{\vec{q}^2 + \lambda_{2n-1}}\\ F^2_{BB_X}(\vec{q}^2) &= \sum_n \frac{g_{v^n}\langle n_{B_X}|\partial_Z\psi_{2n-1}(Z)|n_B\rangle}{\lambda_{2n-1}(\vec{q}^2 + \lambda_{2n-1})}\\ F^3_{BB_X}(\vec{q}^2) &= \sum_n \frac{g_{v^n}\langle n_{B_X}|\partial_Z\psi_{2n-1}(Z)\partial_Z|n_B\rangle}{\lambda_{2n-1}(\vec{q}^2 + \lambda_{2n-1})}\,,\end{aligned} \tag{4.20}$$

the momentum $\vec{q}$ is the photon momentum defined by $\vec{q} = \vec{p}_X - \vec{p}$ and we have used

$$\langle\vec{p}_X|e^{-i\vec{k}\cdot\vec{X}}|\vec{p}\rangle = \delta^3(\vec{k} - \vec{p} + \vec{p}_X)\,. \tag{4.21}$$

The following relations are very useful and are presented here for completeness,

$$\begin{aligned}\langle s_X, I_3^X|s, I_3\rangle_R &= \frac{1}{2\sqrt{E_XE}}(ff_X - \tfrac{ss_X|p||\vec{p}_X|}{ff_X})\,\delta_{I_3^X I}\chi^\dagger_{s_X}(\vec{p_X})\,\chi_s(\vec{p})\,,\\ \langle s_X, I_3^X|_R\,\mathrm{tr}(\tau^c\mathbf{a}\tau_a^{-1}\mathbf{a})|s, I_3\rangle_R &= -\frac{1}{3\sqrt{E_XE}}(ff_X - \tfrac{ss_X|p||\vec{p}_X|}{ff_X})\,\tau^c_{I_3^X I_3}\,\chi^\dagger_{s_X}(\vec{p_X})\,\sigma^a\chi_s(\vec{p})\,,\end{aligned}$$

$$
\begin{aligned}
\langle s_X, I_3^X|_R\, I^c\, |s, I_3\rangle_R &= \frac{1}{4\sqrt{E_X E}}(f f_X - \tfrac{s s_X |p||\vec{p}_X|}{f f_X})\, (\tau^c)_{I_3^X I} \chi^\dagger_{s_X}(\vec{p_X})\, \chi_s(\vec{p})\,, \\
\langle s_X, I_3^X|_R\, S_a\, |s, I_3\rangle_R &= \frac{1}{4\sqrt{E_X E}}(f f_X - \tfrac{s s_X |p||\vec{p}_X|}{f f_X})\, \delta_{I_3^X I} \chi^\dagger_{s_X}(\vec{p_X})\, \sigma_a \chi_s(\vec{p})\,,
\end{aligned}
$$

Positive parity resonances in the Breit frame. In the Breit frame we get for positive parity resonances

$$
\begin{aligned}
\langle J^{0,0}_{V(SS)}(0)\rangle &= \frac{N_c}{2(2\pi)^3}\xi \delta_{I_3^X I}\chi^\dagger_{s_X}(\vec{p_X})\chi_s(\vec{p}) F^1_{BB_X}(\vec{q}^2)\,, \\
\langle J^{i,0}_{V(SS)}(0)\rangle &= \frac{N_c}{2(2\pi)^3 M_0}\delta_{I_3^X I}\chi^\dagger_{s_X}(\vec{p_X})\Big\{ q^i \left[F^3_{BB_X}(\vec{q}^2) - \frac{1}{2x} F^1_{BB_X}(\vec{q}^2)\right]\xi \\
&- \frac{i}{4}\alpha \epsilon^{ija} q_j \sigma_a F^1_{BB_X}(\vec{q}^2)\Big\}\chi_s(\vec{p})\,, \\
\langle J^{0,c}_{V(SS)}(0)\rangle &= \frac{\xi}{2(2\pi)^3}(\tau^c)_{I_3^X I}\chi^\dagger_{s_X}(\vec{p_X})\chi_s(\vec{p}) F^1_{BB_X}(\vec{q}^2)\,, \\
\langle J^{i,c}_{V(SS)}(0)\rangle &= -i\frac{\alpha}{2(2\pi)^3}\left(\frac{M_0}{3}\right)(\tau^c)_{I_3^X I}\langle n_\rho|\rho^2|n_\rho\rangle\, \epsilon^{ija} q_j \chi^\dagger_{s_X}(\vec{p_X})\sigma_a \chi_s(\vec{p}) F^1_{BB_X}(\vec{q}^2)\,,
\end{aligned}
\tag{4.22}
$$

where

$$
\begin{aligned}
\xi &= \left(\frac{1}{2E}\right)\left(\frac{\sqrt{E+m_B}}{\sqrt{E+m_{B_X}}}\right)\left[E + m_{B_X} + (E - m_B)(2x-1)\right]\,, \\
\left(\frac{M_0}{3}\right)\langle n_\rho|\rho^2|n_\rho\rangle &= \frac{1}{\sqrt{6}M_*}\left[1 + 2\sqrt{1+\frac{N_c^2}{5}}\right] \equiv \frac{g_{I=1}}{4m_B}\,.
\end{aligned}
\tag{4.23}
$$

and α is given in (2.21). We have also used the relation

$$
(f f_X - \tfrac{s s_X |p||\vec{p}_X|}{f f_X}) = \frac{f}{f_X}\left[E + m_{B_X} - s s_X (E - m_B)|2x-1|\right]\,, \tag{4.24}
$$

and the identity (2.22) which are valid only in the Breit frame.

4.2. The generalized Dirac and Pauli form factors

Using the holographic prescription (4.2) we can compare, in the case of positive parity baryons, the current matrix element of (2.19),(2.20) with the Sakai-Sugimoto current matrix element in (4.22). As a consequence we get the Dirac and Pauli form factors

$$
F^{D,0}_{BB_X}(q^2) = \left[\frac{\xi\alpha + \beta\alpha\frac{q^2}{4M_0}}{\alpha^2 + \beta^2 q^2}\right] N_c F^1_{BB_X}(q^2)\,, \tag{4.25}
$$

$$
F^{P,0}_{BB_X}(q^2) = -\frac{1}{\kappa_B}\left[\frac{\beta\xi - \frac{\alpha^2}{4M_0}}{\alpha^2 + \beta^2 q^2}\right] N_c F^1_{BB_X}(q^2)\,, \tag{4.26}
$$

$$F^{D,3}_{BB_X}(q^2) = \left[\frac{\xi\alpha + \beta\alpha q^2\left(\frac{M_0}{3}\right)\langle\rho^2\rangle}{\alpha^2+\beta^2 q^2}\right] F^1_{BB_X}(q^2)\,, \tag{4.27}$$

$$F^{P,3}_{BB_X}(q^2) = -\frac{1}{\kappa_B}\left[\frac{\beta\xi - \alpha^2\left(\frac{M_0}{3}\right)\langle\rho^2\rangle}{\alpha^2+\beta^2 q^2}\right] F^1_{BB_X}(q^2)\,, \tag{4.28}$$

where α and β are given in (2.21) and ξ is given in (4.23). Then the electromagnetic Dirac and Pauli form factors read

$$\begin{aligned} F^{D}_{BB_X}(q^2) &= \frac{1}{2}\left[\frac{1}{N_c}F^{D,0}_{BB_X}(q^2) + F^{D,3}_{BB_X}(q^2)\right]\,, \\ F^{P}_{BB_X}(q^2) &= \frac{1}{2}\left[\frac{1}{N_c}F^{P,0}_{BB_X}(q^2) + F^{P,3}_{BB_X}(q^2)\right]\,. \end{aligned} \tag{4.29}$$

4.2.1. The non-relativistic (large λ) limit

First of all note that

$$m_{B_X} = m_B + \frac{2}{\sqrt{6}} n_{B_X} M_*\,, \tag{4.30}$$

so that

$$x = \frac{q^2}{q^2 + \left(2m_B + \frac{2}{\sqrt{6}}n_{B_X}M_*\right)\left(\frac{2}{\sqrt{6}}n_{B_X}M_*\right)}\,. \tag{4.31}$$

Elastic case. In the elastic case we have $n_{B_X}=0$, $m_{B_X}=m_B$ and $x=1$ so in the large λ limit we find

$$\begin{aligned} -\frac{1}{\kappa_B}\left(\beta - \frac{\alpha}{4M_0}\right) &= \frac{m_B}{2M_0} - 1 + \mathcal{O}\left(\frac{1}{\lambda N_c}\right) = \frac{g_{I=0}}{2} - 1 + \mathcal{O}\left(\frac{1}{\lambda N_c}\right)\,, \\ -\frac{1}{\kappa_B}\left[\beta - \alpha\left(\frac{M_0}{3}\right)\langle\rho^2\rangle\right] &= \frac{g_{I=1}}{2}\left[1 + \mathcal{O}\left(\frac{1}{\lambda N_c}\right)\right]\,, \end{aligned} \tag{4.32}$$

so that

$$F^{D,0}_{BB_X}(q^2) = \left[1 + \mathcal{O}\left(\frac{1}{\lambda^2 N_c^2}\right)\right] N_c F^1_{BB_X}(q^2)\,, \tag{4.33}$$

$$F^{P,0}_{BB_X}(q^2) = \left[\frac{g_{I=0}}{2} - 1 + \mathcal{O}\left(\frac{1}{\lambda N_c}\right)\right] N_c F^1_{BB_X}(q^2)\,, \tag{4.34}$$

$$F^{D,3}_{BB_X}(q^2) \;=\; \left[1+\mathcal{O}\left(\frac{1}{\lambda}\right)\right] F^1_{BB_X}(q^2)\,, \tag{4.35}$$

$$F^{P,3}_{BB_X}(q^2) \;=\; \frac{g_{I=1}}{2}\left[1+\mathcal{O}\left(\frac{1}{\lambda N_c}\right)\right] F^1_{BB_X}(q^2)\,, \tag{4.36}$$

Non-elastic case. In the non-elastic case, we have $n_{B_X} = 2, 4, 6, \ldots$, so that in the large λ limit we have

$$\begin{aligned}
\frac{m_B}{E} &= \left(1+\frac{q^2}{4x^2 m_B^2}\right)^{-1/2} = \left(1+\frac{2}{3}\frac{n_{B_X}^2 M_*^2}{q^2}\right)^{-1/2} \sim \mathcal{O}(1)\,,\\
\xi\alpha+\beta\alpha\frac{q^2}{4M_0} &= \frac{m_B}{E}+\mathcal{O}\left(\frac{1}{\lambda N_c}\right)\,,\\
-\frac{1}{\kappa_B}\left(\beta\xi-\frac{\alpha^2}{4M_0}\right) &= \frac{m_B}{2M_0}-\frac{m_B}{E}+\mathcal{O}\left(\frac{1}{\lambda N_c}\right)\\
&= \frac{g_{I=0}}{2}-\frac{m_B}{E}+\mathcal{O}\left(\frac{1}{\lambda N_c}\right)\\
\xi\alpha+\beta\alpha q^2\left(\frac{M_0}{3}\right)\langle\rho^2\rangle &= \frac{m_B}{E}+\mathcal{O}\left(\frac{1}{\lambda}\right)\,,\\
-\frac{1}{\kappa_B}\left[\beta\xi-\alpha^2\left(\frac{M_0}{3}\right)\langle\rho^2\rangle\right] &= \frac{g_{I=1}}{2}\left[1+\mathcal{O}\left(\frac{1}{\lambda N_c}\right)\right]\,.
\end{aligned} \tag{4.37}$$

Thus, in the non-elastic case, we obtain in the large λ limit

$$F^{D,0}_{BB_X}(q^2) \;=\; \left[\frac{m_B}{E}+\mathcal{O}\left(\frac{1}{\lambda N_c}\right)\right] N_c F^1_{BB_X}(q^2)\,, \tag{4.38}$$

$$F^{P,0}_{BB_X}(q^2) \;=\; \left[\frac{g_{I=0}}{2}-\frac{m_B}{E}+\mathcal{O}\left(\frac{1}{\lambda N_c}\right)\right] N_c F^1_{BB_X}(q^2)\,, \tag{4.39}$$

$$F^{D,3}_{BB_X}(q^2) \;=\; \left[\frac{m_B}{E}+\mathcal{O}\left(\frac{1}{\lambda}\right)\right] F^1_{BB_X}(q^2)\,, \tag{4.40}$$

$$F^{P,3}_{BB_X}(q^2) \;=\; \frac{g_{I=1}}{2}\left[1+\mathcal{O}\left(\frac{1}{\lambda N_c}\right)\right] F^1_{BB_X}(q^2)\,. \tag{4.41}$$

4.3. Numerical results

4.3.1. Baryon wavefunctions

Here, we present the results for some low-lying baryon wavefunctions in fig. 2. We restrict our presentation to the parity even baryon wavefunctions $n = 2j$, because, as we shall see below, all coupling constants involving an even baryon state (e.g. the proton) and an odd baryon state will yield zero. This follows directly from vector meson dominance and the fact that vector meson wavefunctions are even.[1] Furthermore, we are only considering baryon states with $(n_\rho)_{\text{initial}} = (n_\rho)_{\text{final}}$, which is zero in the case of the proton; all other possibilities will also produce vanishing results in the calculation of the form factors below. Table 1 summarizes the mass spectrum for the proton $B = (1, +1/2, 0, 0)$ and its excitations

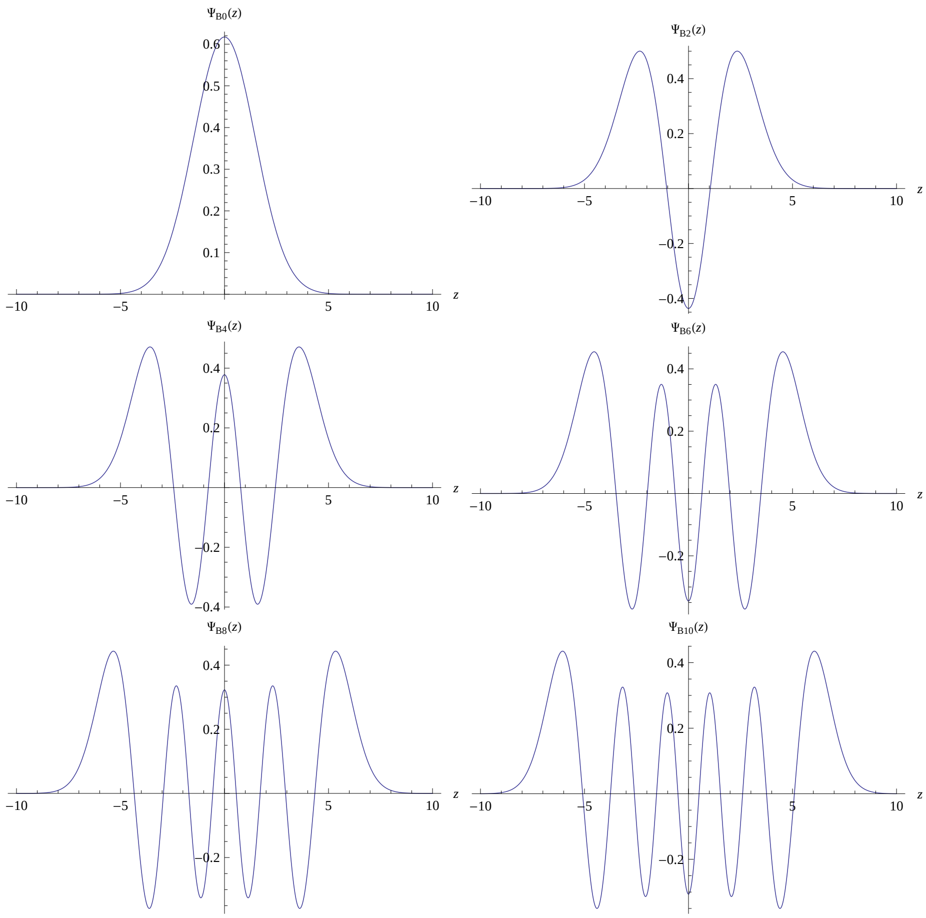

Figure 2: (Normalized) wave functions $\Psi_{B_{2k}}(z)$ for the first six even baryon states.

$B_X = (1, +1/2, 0, n_z)$. Note that we have chosen the proton mass $m_{B_0} = \widetilde{M}_0 = 940$ MeV for obvious phenomenological reasons, although strictly speaking the masses are all proportional to N_c in the holographic limit $N_c, \lambda \to \infty$.[2]

[1] If one picks an odd parity initial baryon state, only odd excited baryon states would contribute to non-vanishing coupling constants.

[2] As noted in [42], changing M_* to a value of 500 MeV would result in a fairly realistic mass spectrum for the excited baryon states, cf. their table (5.35). However, changing M_* would alter the baryon wave functions, vector meson decay constants, coupling constants and so forth. Therefore it is not permissible to merely adapt the mass spectrum by changing M_*.

n	0	1	2	3	4	5	6	7	8
m_{B_n}/GeV	0.940	1.715	2.490	3.265	4.039	4.814	5.589	6.655	7.472

Table 1: Some numerical values for the masses of the excited baryon states connected to the proton.

4.3.2. Baryon form factors

According to figure 1, we need to study the interaction of the baryons with (external) photons. This process is represented by the electromagnetic form factors. The Dirac and Pauli form factors were discussed at length in section 2. Here we present our numerical results for the generalized baryon form factors. The infinite sums over vector meson states appearing in the mathematical description of form factors were approximated by including the first 60 (!) vector mesons states in the numerical computations. The wavefunctions $\psi_{2k-1}(z)$ of the vector mesons were discussed at length in ref. [17].
Some numerical results for these quantities are listed in table 2. With these results we can

k	1	2	3	4	5	6	7	8	9
$\frac{m^2_{v^k}}{M^2_*}$	0.6693	2.874	6.591	11.80	18.49	26.67	36.34	47.49	60.14
$\frac{g_{v^k}}{\sqrt{\kappa}M^2_*}$	2.109	9.108	20.80	37.15	58.17	83.83	114.2	149.1	188.7
$g_{v^k B_0 B_0}$	5.767	-2.610	0.1902	0.7664	-0.5162	-0.01955	0.2118	-0.08413	-0.05348
$g_{v^k B_0 B_2}$	-0.9276	2.4670	-2.6239	1.0560	0.6404	-0.9990	0.2499	0.3848	-0.3049
$g_{v^k B_0 B_4}$	0.3655	-1.2608	2.0708	-1.9409	0.6966	0.6716	-0.9855	0.2490	0.4558
$g_{v^k B_0 B_6}$	-0.1871	0.7299	-1.4596	1.8556	-1.3815	0.1713	0.8448	-0.8371	0.03738
$g_{v^k B_0 B_8}$	0.1091	-0.4595	1.0352	-1.5643	1.5657	-0.7918	-0.3319	0.9318	-0.5634

Table 2: Dimensionless squared masses and decay constants for vector mesons and coupling constants between vector mesons and baryons.

now easily obtain the Dirac and Pauli form factors for the first few baryon states, as shown in fig. 3. Observe that the form factors $F^{D,P}_{B_0B_4}$ are negative definite. This is not problematic since they will only appear in bilinear combinations in the derivation of the structure functions below, so that the sign will not matter.

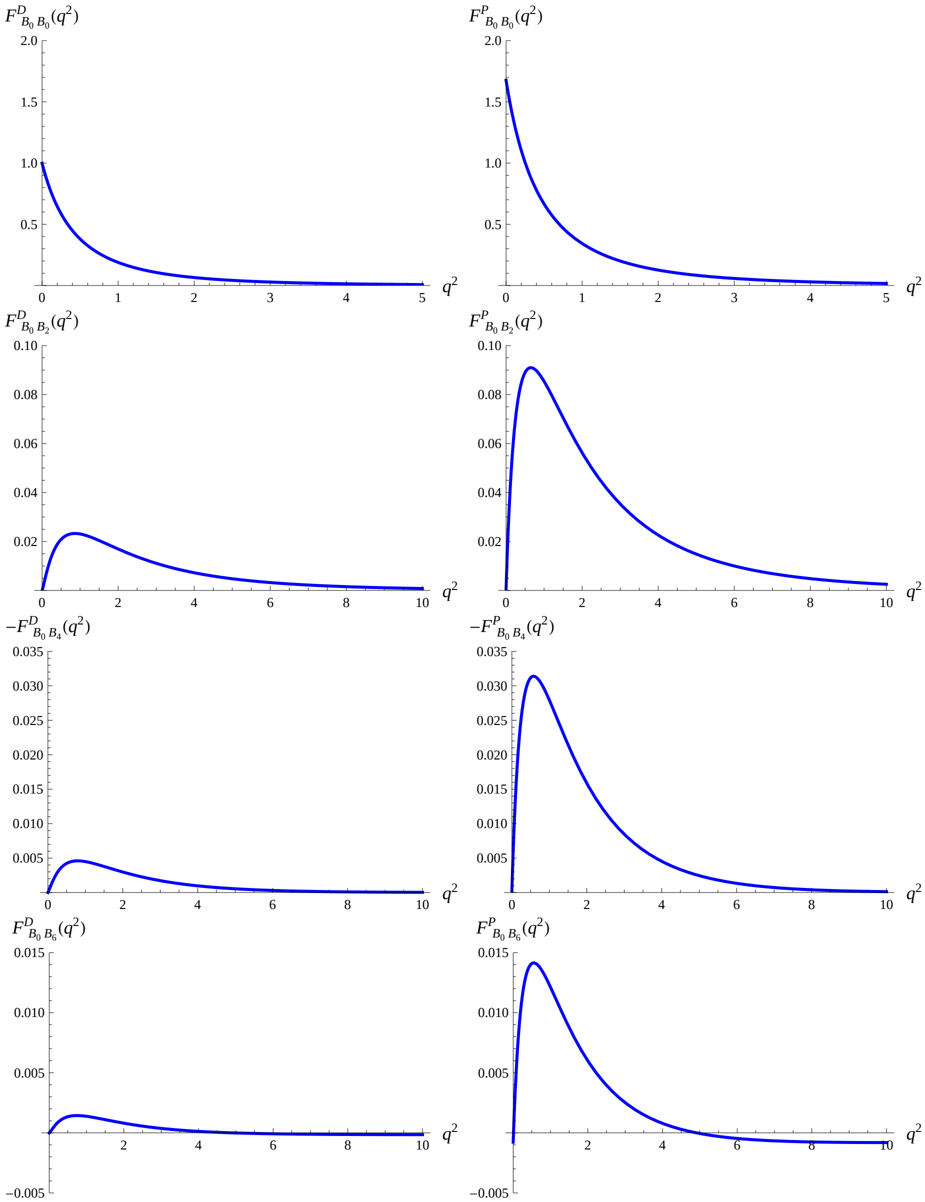

Figure 3: Dirac and Pauli form factors $F^{D,P}_{B_0 B_{2j}}(q^2)$ for the first four baryon states. The elastic case corresponds to $j = 0$, while $j > 0$ yields the transitional form factors. For $j \geq 4$, the numerical errors become relatively large; we can only trust our results for approx. $q^2 \leq 5\,(\mathrm{GeV})^2$.

4.3.3. Helicity amplitudes

In figure 4 we present our numerical results for the helicity amplitudes $G^{+}_{BB_X}(q^2)$ and $G^{0}_{BB_X}(q^2)$ that have been discussed in section 2.2.2. As before, we study the helicity amplitudes in the large λ limit, where the expressions simplify to

$$
\begin{aligned}
(G^+_{BB_X}(q^2))^2 &= \left[\frac{q}{\sqrt{2}m_B}\left(1+\frac{2}{3}n^2_{B_X}\frac{M_*^2}{q^2}\right)^{1/2}(F^D_{BB}(q^2)+F^P_{BB}(q^2))\right]^2 , \\
(G^0_{BB_X}(q^2))^2 &= \left(1+\frac{q^2}{4m_B^2x^2}\right)(F^D_{BB}(q^2))^2 .
\end{aligned}
\tag{4.42}
$$

The same limitations as for the generalized form factors apply here as well, i.e., the numerical errors become significant for $j \geq 4$ and $q^2 \geq 5(\mathrm{GeV})^2$. Therefore the increase observed in $G^+_{BB_X}(q^2)$ for $j = 3, 4$ and q^2 greater than $4(\mathrm{GeV})^2$ may be an artefact of the numerics.

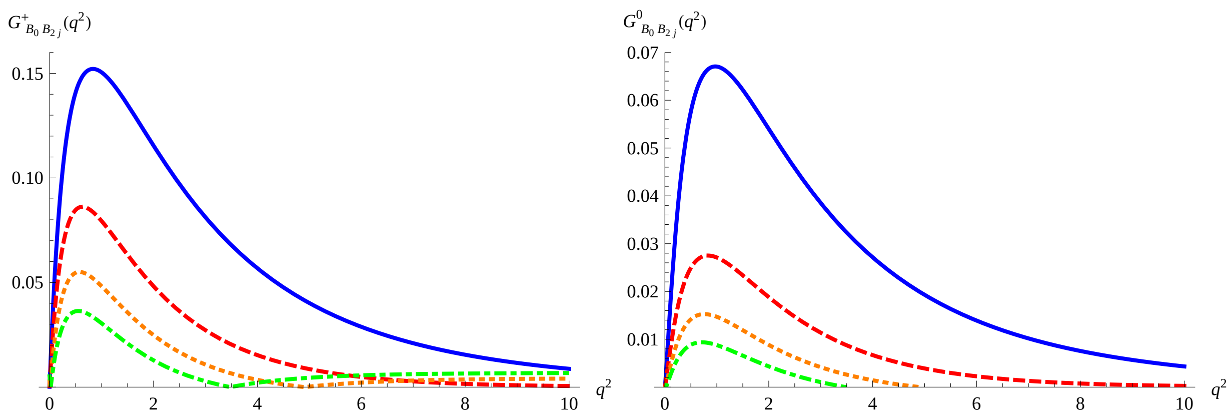

Figure 4: Helicity amplitudes $G^{+;0}_{B_0B_{2j}}(q^2)$ plotted versus q^2 in $(\mathrm{GeV})^2$. The transitions from protons to the baryonic final states are labelled by j, where $j = 1$ (blue, solid), $j = 2$ (red, dashed), $j = 3$ (orange, dotted) and $j = 4$ (green, dotdashed).

5. The proton structure functions

5.1. Baryon Regge trajectories in the Sakai-Sugimoto model

Assuming approximate continuity of the mass distribution, we can now approximate the delta distributions in the following way:

$$
\begin{aligned}
\sum_{B_\mathbf{X}} \delta[m^2_{B_\mathbf{X}} - s] &\equiv \sum_n \delta[m_n^2 - m_{\bar{n}}^2] = \int dn \left[\left|\frac{\partial m_n^2}{\partial n}\right|\right]^{-1} \delta(n-\bar{n}) \\
&= \left[\left|\frac{\partial m_n^2}{\partial n}\right|\right]^{-1}_{n=\bar{n}} \equiv f(\bar{n}),
\end{aligned}
\tag{5.1}
$$

with the definition

$$
s := -(p+q)^2 = m^2_{B_0} + q^2(\frac{1}{x}-1). \tag{5.2}
$$

Therefore we have to evaluate the Regge trajectory of the baryon spectrum in order to calculate $\frac{\partial m_n^2}{\partial n}$. We find (cf. eq. (3.25))

$$\frac{\partial m_n^2}{\partial n} = \left(\frac{4}{\sqrt{6}} \widetilde{M}_0 M_* + \frac{4}{3} n M_*^2 \right) , \tag{5.3}$$

where $\widetilde{M}_0$ can be chosen to match, e.g. the proton mass m_{B_0} and $n := n_z$.

5.2. Numerical results

5.2.1. Dependence of $F_{1,2}$ on q^2 and x

It is now possible to extract information about the structure functions $F_{1,2}(q^2, x)$ employing the following strategy: From the discrete set of baryon mass states and the relation (5.2), we find

$$\Delta m_{B_{2j}}^2 := m_{B_{2j}}^2 - m_{B_0}^2 = q^2 \left(\frac{1}{x} - 1 \right) . \tag{5.4}$$

It is possible for each $j > 0$ to extract a discrete set of values for q^2 for a given set of fixed Bjørken parameters, e.g., $x = 0.001$, 0.01, 0.05, 0.1, 0.3. Alternatively, we may calculate a set of values for x for a given set of, e.g., $q^2 = 0.1$, 0.5, 1, 2, 3 (GeV)^2. The corresponding values are summarized in table 3. Lastly, we need to collect some results about the calculation of the isoscalar and isovector magnetic moments for the states under consideration. For the proton and its excited states with $n_\rho = 0$ and spin up, we find (cf. eqs. (3.16) and (3.32) of [22])

$$\mu_{I=0}^i = \frac{1}{4M_0} \delta^{3i} \approx 0.842 \, \delta^{3i} \, \mu_B, \quad \mu_{I=1}^i = \frac{M_0}{3} \frac{\sqrt{5} + 2\sqrt{5 + N_c^2}}{2N_c} \, \rho_{\rm cl}^2 \, \delta^{3i} \approx 3.52 \, \delta^{3i} \, \mu_B, \tag{5.5}$$

measured in units of the Bohr magneton $\mu_B = 1/(2m_{B_0})$. Here, the mass and size of the classical instanton are $M_0 = 8\pi^2 \kappa M_* = 558$ MeV and $\rho_{\rm cl}^2 = \frac{N_c}{8\pi^2 \kappa M_*^2} \frac{\sqrt{6}}{\sqrt{5}}$, respectively, and we have set $N_c = 3$ for obvious phenomenological reasons. Sometimes it will be more convenient to utilize magnetic g_I factors, which can be defined as follows,

$$\mu_I^i = \frac{g_I}{4m_{B_X}} \sigma^i, \tag{5.6}$$

where σ^i are Pauli matrices. The numerical values in the Sakai-Sugimoto model turn out to be

$$g_{I=0} \approx 1.684 \, , \qquad g_{I=1} \approx 7.031 \, . \tag{5.7}$$

It is now a fairly straightforward exercise to evaluate the structure functions $F_{1,2}(q^2, x)$ for the discrete set of values given in table 3. Again, it should be stressed that we work in the large λ limit, i.e., we only keep the leading terms in the large λ, large N_c expansion *before* setting the baryon masses (which are of order $\mathcal{O}(\lambda N_c)$) to there phenomenological values. The results are presented in figures 5 and 6.

j	$f(\bar{n}=2j)/M_*^2$	$\Delta m^2_{B_{2j}}/(\mathrm{GeV})^2$	$q^2/(\mathrm{GeV})^2$	x
1	0.233	5.317	0.00532 for $x = 0.001$ 0.0537 for $x = 0.01$ 0.280 for $x = 0.05$ 0.591 for $x = 0.1$ 2.279 for $x = 0.3$	0.0185 for $q^2 = 0.1(\mathrm{GeV})^2$ 0.0860 for $q^2 = 0.5(\mathrm{GeV})^2$ 0.158for $q^2 = 1(\mathrm{GeV})^2$ 0.273 for $q^2 = 2(\mathrm{GeV})^2$ 0.361 for $q^2 = 3(\mathrm{GeV})^2$
2	0.144	15.430	0.0154 for $x = 0.001$ 0.156 for $x = 0.01$ 0.812 for $x = 0.05$ 1.714 for $x = 0.1$ 6.613 for $x = 0.3$	0.00644 for $q^2 = 0.1(\mathrm{GeV})^2$ 0.0314 for $q^2 = 0.5(\mathrm{GeV})^2$ 0.0609for $q^2 = 1(\mathrm{GeV})^2$ 0.115 for $q^2 = 2(\mathrm{GeV})^2$ 0.163 for $q^2 = 3(\mathrm{GeV})^2$
3	0.0814	30.353	0.0304 for $x = 0.001$ 0.307 for $x = 0.01$ 1.598 for $x = 0.05$ 3.373 for $x = 0.1$ 13.008 for $x = 0.3$	0.00328 for $q^2 = 0.1(\mathrm{GeV})^2$ 0.0162 for $q^2 = 0.5(\mathrm{GeV})^2$ 0.0319for $q^2 = 1(\mathrm{GeV})^2$ 0.0618 for $q^2 = 2(\mathrm{GeV})^2$ 0.0899 for $q^2 = 3(\mathrm{GeV})^2$
4	0.0436	54.947	0.0550 for $x = 0.001$ 0.555 for $x = 0.01$ 2.892 for $x = 0.05$ 6.105 for $x = 0.1$ 23.549 for $x = 0.3$	0.00182 for $q^2 = 0.1(\mathrm{GeV})^2$ 0.00902 for $q^2 = 0.5(\mathrm{GeV})^2$ 0.0179for $q^2 = 1(\mathrm{GeV})^2$ 0.0351 for $q^2 = 2(\mathrm{GeV})^2$ 0.0518 for $q^2 = 3(\mathrm{GeV})^2$

Table 3: Some values for q^2 and x according to eq. (5.4).

5.2.2. Callan-Gross relation

The Callan-Gross relation can be easily studied numerically in this framework in order to check its validity for spin 1/2 particles, i.e., $F_2/(2xF_1) = 1$. This relation was verified experimentally only for a certain range of values of q^2 and x. The analytic expression for the corresponding ratio of structure functions can be simplified and reads

$$R_{\mathrm{CG}}(q^2,x) := \frac{F_2(q^2,x)}{2x\,F_1(q^2,x)},$$

$$= \frac{m_B + m_{B_X}(q^2,x)}{2x^2 m_B + x(m_{B_X}(q^2,x) - m_B)} \frac{\frac{m_B}{\sqrt{m_B^2+\frac{q^2}{4x^2}}} + \frac{q^2\kappa_B^2}{4}\left(\frac{g_{I=0}+g_{I=1}}{2} - \frac{m_B}{\sqrt{m_B^2+\frac{q^2}{4x^2}}}\right)^2}{\left(\frac{m_B}{\sqrt{m_B^2+\frac{q^2}{4x^2}}} + \frac{1}{2}\left(\frac{g_{I=0}+g_{I=1}}{2} - \frac{m_B}{\sqrt{m_B^2+\frac{q^2}{4x^2}}}\right)\right)^2}. \quad (5.8)$$

Note that the dependence on the Dirac and Pauli form factors drops out of the equation completely. Here, m_B is taken to be the mass of the proton as above, and again, we will utilize the relation (5.4) to plot $R_{\mathrm{CG}}(x)$ for several fixed values of q^2. The result is presented in figure 7. We find that the ratio $R_{\mathrm{CG}}(x)$ diverges for small x, but asymptotes to one for intermediate values of x that are still in the range of validity of our model.

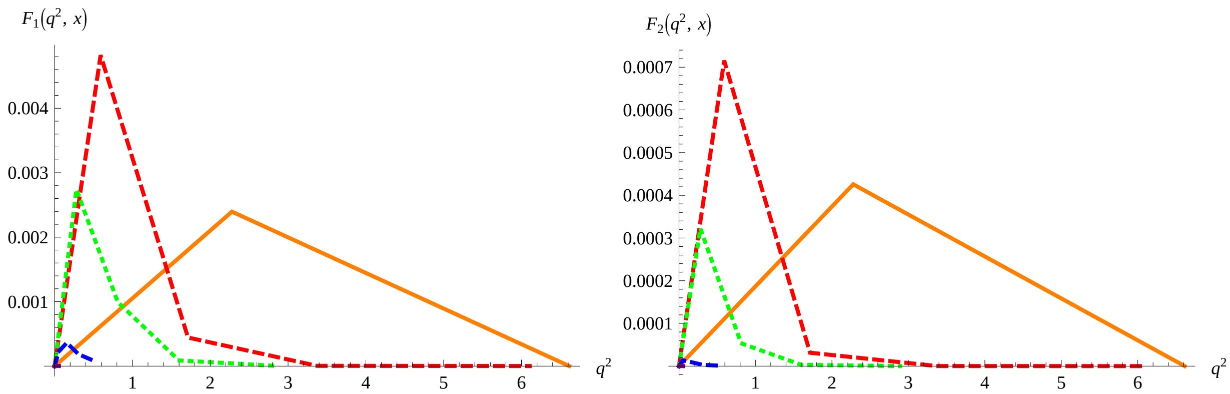

Figure 5: Structure functions $F_{1,2}(q^2)$ for $x = 0.3$ (orange, solid), $x = 0.1$ (red, dashed), $x = 0.05$ (green, dotted) and $x = 0.001$ (blue, dotdashed, barely visible).

6. Conclusions and Outlook

In the present paper we have derived a generalization of the notions of baryon electromagnetic form factors to non-elastic scattering when baryonic resonances with positive parity are produced. Subsequently, we have used the Sakai-Sugimoto model to estimate these form factors in the non-perturbative regime of large N_c QCD. We have also estimated the contribution of these form factors to the proton structure functions. The results for the proton structure functions obtained herein are understood to be non-inclusive and only represent a small fraction of possible final states, namely single final state baryons (the excited states of the proton) with spin 1/2 and positive parity. Therefore the magnitude of $F_{1,2}(q^2, x)$ is at least two orders of magnitude smaller than what is expected from experimental results for the full inclusive process of DIS (cf., e.g., [47], chapter 16). Contribution from final states with spin 1/2 and negative parity [48] as well as final states with higher spin[3] and pion production may be relevant to get a better picture of the proton structure functions.

It is important to remark that, in this paper, we have considered the large λ limit in the Sakai-Sugimoto model where the description of baryons as solitons is derived from classical small instanton solutions, whose size is of order $\rho^2 \sim \lambda^{-1/2}$. There may exist $1/\lambda$ corrections that can be dominant at large distances as suggested in a recent analysis [49,50]. One should also bear in mind the limitations of describing baryons in the large N_c limit [4]. Interestingly, in [52] holographic baryons were constructed for the (bottom-up) hard wall model as solitons in an AdS_5 spacetime with cut-off. These baryons have finite size and have the expected long-distance properties [49]. It would be interesting to investigate baryon resonance production in this model and recent holographic models such as [53,54] or the recent string models based on the singular [55,18] and deformed [56,57] conifold backgrounds. It would be also

[3]See [24] for transition to Δ resonances in the Sakai-Sugimoto model

[4]There occurs a phase transition at $N_c \approx 8$ that separates the small N_c ($= 3$) regime where nuclear matter behaves like a quantum liquid from the large N_c (holographic) regime where nuclear matter behaves like a crystalline solid [51]

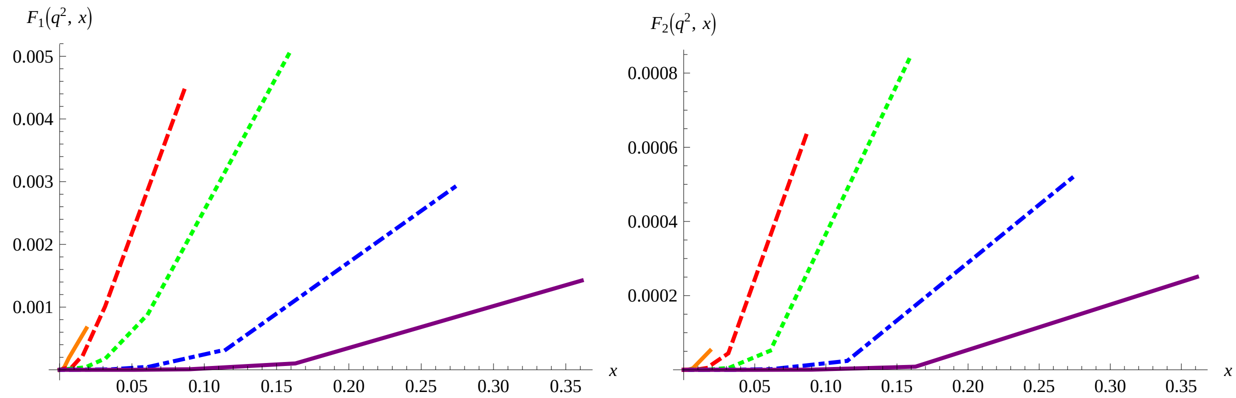

Figure 6: Structure functions $F_{1,2}(x)$ for $q^2 = 3(\text{GeV})^2$ (purple, solid), $q^2 = 2(\text{GeV})^2$ (blue, dotdashed), $q^2 = 1(\text{GeV})^2$ (green, dotted), $q^2 = 0.5(\text{GeV})^2$ (red, dashed) and $q^2 = 0.1(\text{GeV})^2$ (orange, solid, barely visible) .

interesting to estimate the contribution of resonance production in other scattering processes like dilepton production in proton-proton scattering (see [59]).

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A. Breit frame for inelastic scattering

Consider the scattering between a virtual photon and a hadron in the hadron rest frame. After two rotations we can set the spatial momentum of the photon to the x^3 direction, so that

$$\begin{aligned} p^\mu &= (m_B, 0, 0, 0) \\ q^\mu &= (q_0, 0, 0, q_3)\,, \end{aligned} \tag{A.1}$$

and we choose $q_3 > 0$. The virtuality and Bjorken variable in this frame are given by

$$Q^2 = q_3^2 - q_0^2 \quad , \quad x = -\frac{Q^2}{2m_B q_0}\,. \tag{A.2}$$

Now we perform a boost in the x^3 direction so that

$$p'^\mu = (\gamma m_B, 0, 0, -\beta\gamma m_B)$$

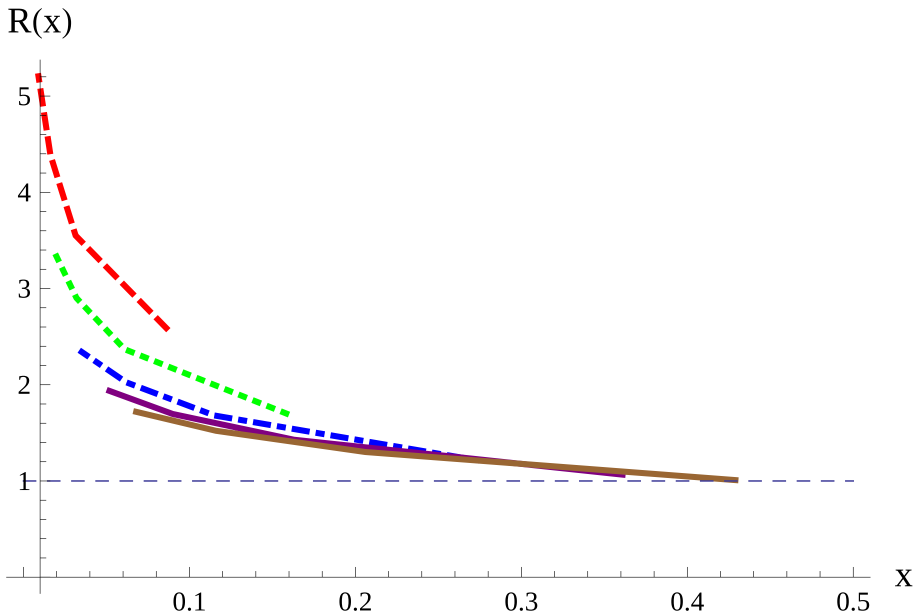

Figure 7: Callan-Gross ratio $R_{\text{CG}}(x)$ for $q^2 = 4(\text{GeV})^2$ (brown, solid), $q^2 = 3(\text{GeV})^2$ (purple, solid), $q^2 = 2(\text{GeV})^2$ (blue, dotdashed), $q^2 = 1(\text{GeV})^2$ (green, dotted) and $q^2 = 0.5(\text{GeV})^2$ (red, dashed).

$$q'^{\mu} = (\gamma q_0 - \beta\gamma q_3, 0, 0, -\beta\gamma q_0 + \gamma q_3)\,. \tag{A.3}$$

The Breit frame is defined by the condition $q'_0 = 0$ so that

$$\beta = \frac{q_0}{q_3} = \frac{q_0}{\sqrt{q_0^2 + Q^2}} \quad , \quad \gamma = \frac{\sqrt{q_0^2 + Q^2}}{Q} \quad , \quad q'_3 = Q\,, \tag{A.4}$$

and we arrive at

$$\begin{aligned} p'^{\mu} &= (\sqrt{m_B^2 + p_3^2}, 0, 0, p_3) \\ q'^{\mu} &= (0, 0, 0, Q)\,, \end{aligned} \tag{A.5}$$

with

$$p_3 = -\frac{Q}{2x}\,. \tag{A.6}$$

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