Entropy and General Equilibrium

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Abstract

The concept of general equilibrium in economics has much in common with that of thermal equilibrium in physics. Entropy is a physical quantity which plays a fundamental rôle in the understanding of thermal equilbrium, and we assert that an analogous quantity, utility, exists in general equilibrium. The thermodynamic formalism is interpreted in the context of general equilibrium, and a simple economic model is presented for illustration.

Introduction

The general equilibrium model of the economy is the product of over two centuries of innovative and intellectual refinement, dating from the time of Smith (1976). In economics general equilibrium analysis refers to the study of the price vector which equates supply with demand on every market in an economy. The solution to the general equilibrium model is the determination of prices needed to equate supply with demand in each market in the economy. The model embodies an *equilibrium* notion in that supply is equal to demand on every market, and is *general* in the sense that the interaction of *all* markets is examined, each simultaneously attaining an equilibrium state.

Léon Walras was the first to provide a general description of the functioning of a complex economic system based on the interaction and interrelationships between a large number of economic agents. It was soon realised that only a mathematical model could fully account for the interdependence of the variables involved. Equilibria are the solutions to sets of equations where supply is equated with demand. In addition to the existence issue, several other questions of mathematical relevance can be examined, such as determinateness and the multiplicity of equilibria. Such concepts require answers to questions of a pure mathematical nature while still retaining relevance for economics. Several mathematical approaches to general equilibrium theory have been identified: the Arrow & Debreu (1954) paper embodied an axiomatic approach relying heavily on the mathematics of fixed point theorems, convexity theory and basic point set topology. The differential approach associated with Smale, Debreu, Balasko, Mas-Colell (and others) attempted to study the structure of the equilibrium set, and study the dependence of the equilibrium price vector on the parameters defining the economy. This was achieved by making differentiability assumptions on the functions entering into the description of the economy (for example demand functions were assumed to be of class C^{∞}). At all times the nature of the economic problem under investigation dictated the employment of the appropriate mathematical tool — it was not the case of arbitrarily choosing a mathematical concept and looking for applications.

The omnipresence of the equilibrium concept in economics is easy to substantiate. It is almost impossible to evade the equilibrium concept. Governments speak of 'equilibrium exchange rates',

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1

banks speak of targeted 'equilibrium rates of interest'. Equilibrium concepts also have a role to play in physics. In particular, thermal equilibrium with the application of *thermodynamics* to study the global properties of equilibrium states has played a dominant role in physics. Consider the definition of *equilibrium* given in Callen (1985, page 13):

> In all systems there is a tendency to evolve toward states in which the properties are determined by intrinsic factors and not by previously applied external influences. Such simple terminal states are by definition, time independent. They are called equilibrium states.

The most striking feature of this definition is its apparent applicability to both physical and economic phenomena. It sounds like a definition in which individuals are maximising preferences (intrinsic factors) irrespective of past events such as periods of high marginal tax rates (external effects). Thermodynamics describes these simple static states to which macroscopic systems eventually evolve, without any assumptions about their microscopic structure. The basic problem of thermodynamics is the calculation of the equilibrium values of the parameters of a system.

Given the above comments it is natural to postulate on the nature of the relationship between the equilibrium concept used by economists and that used by physicists.

Physics has often been used, as in Malchup (1978), as a benchmark to evaluate the scientific status (if any) of economics, as it is believed to attain a level of purity, precision, and objectivity not achieved in the social sciences. The success of thermodynamics in analysing the equilibrium states of physical systems is impressive, and makes one wonder if such techniques can have similar success when applied to phenomena in economic equilibria. It is believed here that this is indeed the case.

Section I outlines the physics used and explains briefly the physical concepts involved, such as thermal equilibrium and entropy. Section II makes the assertion that a 'global utility' function can be identified with entropy. Section III derives a functional form for the entropy of a simple economic model and discusses the nature of the results obtained.

Section I

Thermodynamics

Thermodynamics is the branch of physics that was born from the study of heat and temperature. In trying to make our intuition about them precise, and explain and predict experimental results, nineteenth century scientists created a theory of remarkable generality and profundity. Thermodynamics makes no assumptions about the microscopic nature of matter, but deals only with its macroscopic properties making predictions on the basis of a few postulates distilled from, and thoroughly verified by, observation and experiment.

The fundamental concept of thermodynamics is thermal equilibrium: any isolated physical system will eventually settle into a state in which the local densities of the physical quantities of interest (such as matter and energy) do not change, and there is no flux of these quantities between different parts of the system. These states are characterised by a uniform temperature throughout the system, and are known as thermal equilibrium states. Thermodynamics does not deal with the processes which bring equilibrium about, nor with how a system moves from one equilibrium state to another, it is a theory of comparative statics. Although there are many possible states of a physical system, and typically states are specified by an enormous number of coordinates, the equilibrium states can be parametrised by just a few. For example, in order to specify completely the state of one litre of gas one must determine the position, velocity and internal state of approximately 10^{22} gas molecules.

In contrast, the equilibrium state can be specified by just three parameters: the total energy of the gas, the volume it occupies, and its mass. The required parameters are the quantities of conserved substances in the system, such as total energy, electric charge and magnetic moment. These are also known as extensive quantities, because if we create an exact replica of our system and combine it with the original, we will have twice as much of each of them as before.

Entropy

Entropy is a quantity associated only with equilibrium states which is as fundamental a physical reality as energy or electric charge, and is substance-like (Falk, Hermann & Schmid 1983), in that it is extensive and has a well-defined density which satisfies a continuity equation. It can be thought of as a fluid with the properties which our intuition ascribes to heat: it can be stored in material bodies, the higher the temperature of a body the more entropy it contains, and entropy flows spontaneously from a hotter body to a cooler one and one has to do work to make it flow the other way (Fuchs 1987). Like heat, but unlike a material fluid, it can be created, by friction for example, but not destroyed. This partial conservation of entropy is closely related to the spontaneity and reversibility of physical processes. Spontaneous processes in an isolated system are those that bring the system to a state of higher entropy. These are irreversible since they create entropy, which cannot be destroyed to restore the original state, while reversible processes are those that do not change the entropy of the system. The Second Law of Thermodynamics summarises the rôle of entropy in thermodynamics:

The state a system will attain in equilibrium is the state for which the entropy is a maximum, subject to the constraints on the system.

The entropy of a system depends on the equilibrium state, and since the state is completely specified by the conserved extensive quantities in the system, we can write the entropy as a function of the latter. In every physical system examined experimentally it has been found that this function is concave, homogeneous of degree 1 and smooth (C^1). The various partial derivatives of this function are quantities known as intensities which have independent physical meaning, and it is interesting to look at the Second Law in terms of them. For example, the entropy S, of a quantity of a pure gas is a function of its energy U, volume V and mass M, which, because of its homogeneity, can be written

$$S(U, V, N) = \frac{\partial S}{\partial U} U + \frac{\partial S}{\partial V} V + \frac{\partial S}{\partial M} M.$$

The partial derivatives are then related to the temperature T, the mechanical pressure p and the chemical potential μ (a quantity important in chemical reactions) as follows:

$$\frac{\partial S}{\partial U} = \frac{1}{T}, \qquad \frac{\partial S}{\partial V} = \frac{p}{T}, \qquad \frac{\partial S}{\partial M} = \frac{\mu}{T}.$$

Two bodies of gas can be brought into thermal contact by putting them into a container with a conducting partition which separates them, but allows energy to flow between them. According to the Second Law, the total entropy of the combination will be maximised subject to the constraint that the total energy be conserved:

$$S^{(t)} = \sup \left\{ S^{(1)} + S^{(2)} \mid U^{(1)} + U^{(2)} = U^{(t)} \right\}$$

This supremum is attained where

$$\frac{\partial S^{(1)}}{\partial U^{(1)}} = \frac{\partial S^{(2)}}{\partial U^{(2)}},$$

that is, where $T^{(1)} = T^{(2)}$: the two systems will be at the same temperature in thermal equilibrium. Similarly, two bodies of gas can be brought into thermal and mechanical contact by putting them into

a container with a movable conducting piston between them, allowing their volumes to change. The Second Law now states that the total entropy will be maximised subject to the constraints that the total energy and the total volume be constant:

$$S^{(t)} = \sup \{ S^{(1)} + S^{(2)} \mid U^{(1)} + U^{(2)} = U^{(t)},\ V^{(1)} + V^{(2)} = V^{(t)} \}$$

The supremum is attained where

$$\frac{\partial S^{(1)}}{\partial U^{(1)}} = \frac{\partial S^{(2)}}{\partial U^{(2)}}, \qquad \frac{\partial S^{(1)}}{\partial V^{(1)}} = \frac{\partial S^{(2)}}{\partial V^{(2)}},$$

that is, where $T^{(1)} = T^{(2)}$ and $p^{(1)} = p^{(2)}$: the two systems will be at the same temperature and pressure in equilibrium. Note that if initially they are at different temperatures or pressures, then the final total entropy will be greater than that initially: entropy is created by bringing systems of different temperature or pressure into direct physical contact.

Gibbsian Thermodynamics

Gibbs (1873) formalised thermodynamics by proposing that any function of the energy, volume and other state variables of a given system, which has the three universal properties of entropy is a valid entropy function. In modern terminology (Gross 1982) these properties are:

1. concavity

2. homogeneity of degree 1

3. smoothness (C^1)

The intensities of the system are now defined to be the partial derivatives of the entropy with respect to the appropriate extensive variables, so, for example, the temperature and pressure are defined by

$$\frac{1}{T} := \frac{\partial S}{\partial U} \qquad p := T \frac{\partial S}{\partial V}.$$

Thus if we are given such an entropy function we can determine quantitatively all the equilibrium properties of the system.

Statistical Mechanics

Statistical mechanics gives a rational explanation of the postulates of thermodynamics on the basis of the microscopic structure of matter, and shows that thermodynamic properties are essentially statistical in nature. In general, there are many microscopic states of a physical system in which the thermodynamic quantities have values corresponding to a given macroscopic equilibrium state. On the macroscopic length scale we cannot distinguish between these microstates, and the time scale of molecular dynamics is so short that, during the time it takes us to make a macroscopic measurement, the system will have been in each of the microstates many times. The fundamental tenet of statistical mechanics is that, in equilibrium, the system is equally likely to be in any of the possible microstates, and is known as the axiom of *equal a priori probabilities*. Boltzmann's insight was that the entropy of an equilibrium macrostate is proportional to the logarithm of the number of corresponding microstates, hence the Second Law of Thermodynamics: if two equilibrium macrostates of different entropy are consistent with the constraints on a system, then there are exponentially more microstates corresponding to the state of higher entropy; since the system spends equal time in all

the possible microstates, it will nearly always be in that macrostate, and effectively it will be the only one we ever observe.

These intuitive ideas are made precise by making a stochastic model of the system: the macroscopically observable quantities are real valued random variables over the microscopic state space, the underlying probability space, and the entropy gives the large deviation properties of their distributions on the volume scale. This means that for large volume V the probability of the densities of the energy $\mathcal{H}$ and the particle number $\mathcal{N}$ lying in a certain range $\mathcal{B}$ is given by the entropy S as follows:

$$\log \mathbb{P}[\left(\frac{\mathcal{H}}{V}, \frac{\mathcal{N}}{V}\right) \in \mathcal{B}] \sim \sup_{(\frac{U}{V}, \frac{N}{V}) \in \mathcal{B}} S(U, V, N) - C,$$

where C is the normalisation factor that makes $\mathbb{P}$ a probability measure.

By choosing a particular model for the state space we can calculate the entropy using the techniques of Large Deviation theory, the mathematical core of statistical mechanics. We must be careful though: if we neglect one of the conserved extensive quantities, X say, in the analysis, then the calculated entropy will differ from the empirical measured entropy S by a term $\frac{\partial S}{\partial X} X$, and our predictions based on it will be at variance with experimental results.

Section II

Utility and Equilibrium

The representation of preferences by a real valued utility function is nearly as old as general equilibrium theory itself. Debreu (1954) derives conditions under which a concave utility function, representing the consumer's preference preordering, exists. The acquisition of utility, or satisfaction, through consumption is the force that drives trade: consumers engage in trade only if it increases their level of utility.

Let us explore the rôle of utility in equilibrium: consider two agents, A and B, interested in trading a good X for money Y, and assume for simplicity that each agent derives the same constant marginal utility λ from money. Thus the functional form of their utilities is

$$u_A = u_A(x_A) + \lambda y_A$$
$$u_B = u_B(x_B) + \lambda y_B,$$

If agent A receives a small quantity Δx of the good in return for a small payment Δy, then, to second order in Δx and Δy, the corresponding changes in utility are

$$\Delta u_A = \frac{\partial u_A}{\partial x_A} \Delta x_A + \frac{1}{2} \frac{\partial^2 u_A}{\partial x_A^2} (\Delta x_A)^2 - \lambda \Delta y_A$$

$$\Delta u_B = -\frac{\partial u_B}{\partial x_B} \Delta x_B + \frac{1}{2} \frac{\partial^2 u_B}{\partial x_B^2} (\Delta x_B)^2 + \lambda \Delta y_B$$

Each agent will strive to maximise his utility subject to his budget constraint, and equilibrium is achieved where the changes in utility are zero to first order in Δx and Δy, and is stable where the changes are negative to second order:

$$\frac{\partial u_A}{\partial x_A} = \lambda = \frac{\partial u_B}{\partial x_B} \qquad \text{(First order condition)}$$

$$\frac{\partial^2 u_A}{\partial x_A^2} < 0, \quad \frac{\partial^2 u_B}{\partial x_B^2} < 0 \qquad \text{(Second order conditions)}$$

5

The total utility, an unweighted sum of individual utilities $U = u_A + u_B$, arises very naturally in this analysis: these equilibrium conditions are also the conditions for U to be locally maximised subject to the constraints that the total quantities X_k of good k and money are constant. This characterisation of stable equilibria generalises to any number of agents trading simultaneously, and is very similar in form to the Second Law of Thermodynamics, suggesting that utility is the analogue of entropy in general equilibrium. An initial difficulty with such an analogy is that the total utility is a function of the microstate of the economy, the holdings of each agent, but entropy is a function of the equilibrium state. However, the total utility is maximised in equilibrium, and the equilibrium states are parametrised by just a few variables: the quantities that are conserved by trade, such as the total holdings of each good and the number of agents. Thus its equilibrium value can be considered to be a function of these variables, a function we shall call the *global utility indicator*. If a number of equilibrium states are consistent with the constraints on the system, then the state the system will attain is the one for which the total.utility, and hence the global utility indicator, is a maximum.

The various partial derivatives of a utility function are identified with the relative prices of goods, and the price

$$p_k = \frac{\partial U}{\partial X_k}$$

of any good k decreases with increasing supply X_k,

$$\frac{\partial p_k}{\partial X_k} \leq 0$$

This, in conjunction with the assumption of diminishing marginal rates of substitution, means that utility functions, and so also the global utility indicator, are concave. Consider an economy which has reached equilibrium: if we create an exact replica of it, the two systems are, by definition, in equilibrium with each other. Therefore if they are brought into contact and allowed to trade, nothing happens, and each agent's individual utility remains unchanged. Since the total utility is additive, and the two economies are identical, its equilibrium value in the combined system is twice that in the original, and hence the global utility indicator is a homogeneous function of degree one.

'Gibbsian' Economics

The global utility indicator has nearly all the properties of an entropy function: it characterises general equilibrium in an economy, and is concave and homogeneous of degree one. The only other property it must have in order to satisfy Gibbs' axioms for entropy is smoothness. The Stone–Weierstrass Theorem tells us that any continuous function can be arbitrarily closely approximated by a smooth one, and we know that the global utility indicator is continuous, since it is concave. The smoothness of the global utility indicator is thus a modest assumption, and a very fruitful one for it allows us to apply the powerful conceptual framework of thermodynamics.

We make this assumption, and adopt Gibbs' formalism:

Proposition: *The global utility indicator of an economy is its Gibbsian entropy.*

The price p_k of each good k is defined in terms of the entropy S:

$$p_k := \frac{\partial S}{\partial X_k}$$

This indicates that price is only meaningful in equilibrium, and this is indeed the case: if agents are trading in good X at different local marginal rates of substitution throughout the economy then it does not make sense to talk about the price of X.

If we can determine the functional form of the utility then we can calculate the relationship between prices, supply of each good and the number of agents in the economy. We can also study what happens when two economies, initially isolated and in equilibrium, are brought into trading contact. Their total entropy S_t is maximised subject to the trading constraints, which, for a pure exchange economy, will include the conservation of the total quantity of each of l goods. If these are the only constraints, then the equilibrium condition is

$$S^{(t)} = \sup \left\{ S^{(1)} + S^{(2)} \mid X_1^{(1)} + X_1^{(2)} = X^{(t)}, \ldots, X_l^{(1)} + X_l^{(2)} = X_l^{(t)} \right\}$$

This supremum is attained when

$$\frac{\partial S^{(1)}}{\partial X_1^{(1)}} = \frac{\partial S^{(2)}}{\partial X_1^{(2)}}, \quad \cdots \quad , \frac{\partial S^{(1)}}{\partial X_l^{(1)}} = \frac{\partial S^{(2)}}{\partial X_l^{(2)}},$$

that is, when the price of each good is the same in both economies, and the solution to these equations will show how much trade must take place in order to bring equilibrium about. If these conditions hold initially then no utility can be gained and no trade takes place, so utility is only created by trade between economies with different price structures. *The ability of thermodynamics to explore the interaction of two or more systems is perhaps the most important feature of relevance to general equilibrium analysis* — after all, general equilibrium is concerned with the equilibrium outcomes of interactions between markets and economies.

Section III

A Simple Economic Model

So far we have only been able to make qualitative predictions about general equilibrium based on the Gibbsian formulation, but in order to obtain quantitative results we need an explicit functional form for the entropy. In physics this information can be obtained from well designed and controlled experiments — a luxury not available to economists. In the absence of adequate data the economist has little choice but to construct models to explain economic phenomena. Entropy functions can be derived from such models using the techniques of statistical mechanics. As an illustration of the method we describe a simple model of a pure exchange economy, and calculate its entropy. We only give an outline of the derivation as we are only interested in the theory of large deviations as a tool, not as a subject per se.

Our model consists of a variable number $M \in \mathbb{N}$ of agents trading l goods $X, \ldots, Z$ without restriction among themselves. The quantity of each good that an agent can hold is a number in E, a subset of $\mathbb{R}_+$. The microscopic state ω of the economy is completely determined by the consumption $(x_i, \ldots, z_i) \in E^l$ of each agent i, and so the state space Ω_M is E^{lM}. The simplicity of the model is that the only conserved extensive quantities are the total supply of each good,

$$X_M := \sum_{i=1}^{M} x_i, \quad \cdots \quad , Z_M := \sum_{i=1}^{M} z_i$$

and the number of agents, M. These parameters suffice to specify the equilibrium state of the economy, and so the global utility indicator is a function of these parameters alone: $S = S(X, \ldots, Z, M)$.

We build the axiom of equal a priori probabilities, adopted from statistical mechanics, into our model by choosing a uniform measure ρ_M on Ω_M, giving equal average occupation time to each microstate. Since Ω_M is the product space E^{lM} we take ρ_M to be the product measure μ^{lM}, where μ

is a uniform measure on E. We construct $\mathbb{K}_M$, the unnormalised distribution on $\mathbb{R}_+^l$ of the average holdings of each good:

$$\mathbb{K}_M[\mathcal{B}] := \rho_M[\{\omega \in \Omega : \left(\frac{X_M(\omega)}{M}, \ldots, \frac{Z_M(\omega)}{M}\right) \in \mathcal{B}\}],$$

for each measurable subset $\mathcal{B}$ of $\mathbb{R}_+^l$. Large Deviation theory tells us that the leading asymptotic behaviour of $\mathbb{K}_M$ for large M is given by its Lanford–Ruelle function s:

$$\lim_{M \to \infty} \frac{1}{M} \log \mathbb{K}_M[\mathcal{B}] = \sup_{(x,\ldots,z) \in \mathcal{B}} s(x, \ldots, z),$$

and the global utility of our model is this function homogenised by M:

$$S(X, \ldots, Z, M) = M s\left(\frac{X}{M}, \ldots, \frac{Z}{M}\right).$$

To calculate s we define the grand canonical pressure c on $\mathbb{R}^{l*}$, the vector dual space of $\mathbb{R}^l$:

$$c(u, \ldots, w) := \lim_{M \to \infty} \frac{1}{M} \log \int_{\mathbb{R}_+^l} e^{uX_M + \ldots + wZ_M} d\mathbb{K}_M$$

It follows from the definition of $\mathbb{K}_M$ and the product form of ρ_M that

$$\int_{\mathbb{R}_+^l} e^{uX_M + \ldots + wZ_M} d\mathbb{K}_M = \int_{\Omega_M} e^{uX_M(\omega) + \ldots + wZ_M(\omega)} \rho_M[d\omega]$$

$$= (\int_E e^{ux} \mu[dx])^M \times \ldots \times (\int_E e^{wz} \mu[dz])^M$$

and so

$$c(u, \ldots, w) = g(u) + \ldots + g(w),$$

where

$$g(v) := \log \int_E e^{vy} \mu[dy].$$

The Lanford function s is the Legendre transform c^* (Rockafellar 1970) of the grand canonical pressure c:

$$
\begin{aligned}
c^*(x, \ldots, z) &:= - \sup_{(u,\ldots,w) \in \mathbb{R}^{l*}} \{ux + \cdots + wz - c(u, \ldots, w)\} \\
&= - \sup_{u \in \mathbb{R}^*} \{ux - g(u)\} - \ldots - \sup_{w \in \mathbb{R}^*} \{wz - g(w)\} \\
&= g(u_x) - x u_x + \ldots + g(w_z) - z w_z,
\end{aligned}
$$

where u_x is the unique solution of $\dfrac{dg}{du}(u) = x$.

To complete the calculation we must choose E and an appropriate measure μ.

I. Consider the case of infinitely divisible goods, as is usually assumed, such as in Arrow & Debreu (1954): E is $\mathbb{R}_+$, and μ is $\mathbb{L}$eb, Lebesgue measure.

$$
\begin{aligned}
g(v) &= \log \int_0^\infty e^{vy} dy \\
&= \begin{cases} -\log(-v) & v < 0 \\ +\infty & v \geq 0 \end{cases}
\end{aligned}
$$

8

and Legendre transform of g is

$$g^*(y) = \log(y) + 1.$$

Thus

$$s(x, \ldots, z) = \log(x \ldots z) + l,$$

and the global utility is

$$S(X, \ldots, Z, M) = X\left(\frac{M}{X}\right) + \ldots + Z\left(\frac{M}{Z}\right) + M\log\left(\frac{X}{M}\right) \ldots \left(\frac{Z}{M}\right).$$

The price p_Y of good Y is

$$p_Y(X, \ldots, Z, M) := \frac{\partial S}{\partial Y} = \frac{M}{Y}.$$

II. Now consider the case where goods exist only in discrete units: E is $\mathbb{N}$, and the appropriate measure μ is counting measure.

$$
\begin{aligned}
g(v) &= \log \sum_{b=0}^{\infty} e^{vb} \\
&= \begin{cases} -\log(1 - e^v) & v < 0 \\ +\infty & v \geq 0 \end{cases}
\end{aligned}
$$

and the Legendre transform of g_μ is

$$g^*(y) = (y + 1)\log(y + 1) - y\log(y),$$

So we have

$$s(x, \ldots, z) = (x + 1)\log(x + 1) - x\log(x) + \ldots + (z + 1)\log(z + 1) - z\log(z),$$

and the global utility is

$$S(X, \ldots, Z, M) = X\log\left(1 + \frac{M}{X}\right) + \ldots + Z\log\left(1 + \frac{M}{Z}\right) + M\log\left(\frac{X}{M} + 1\right) \ldots \left(\frac{Z}{M} + 1\right).$$

The price p_Y of good Y is

$$p_Y(X, \ldots, Z, M) := \frac{\partial S}{\partial Y} = \log\left(1 + \frac{M}{Y}\right).$$

Features of the model

For high average holdings, $X \gg M, \ldots, Z \gg M$, the entropy in each case is asymptotically the same, which we expect as the granular nature of the holdings in the discrete model becomes indiscernible in this limit. However the entropy of the continuous model is logarithmically singular whenever the supply of any good is zero, while in the discrete case the entropy is finite everywhere on $\mathbb{R}_+^l$, and attains its infimum, zero, when the supply of every good is zero. The Third Law of Thermodynamics, also known as Nernst's Postulate, states that the entropy of any physical system is zero at zero absolute temperature. Since price is the analogue in economics of inverse temperature,

$$p_Y = \frac{\partial S}{\partial Y} \qquad , \qquad \frac{1}{T} = \frac{\partial S}{\partial S}$$

and in both cases of our model
$$\lim_{Y \to 0} p_Y = \infty,$$
the discrete case satisfies Nernst's Postulate, while the continuous case does not, suggesting that the former is more reasonable.

A limitation of our model is that in both cases the price of each good depends only on the supply of that good, an unrealistic result which is due to the simplicity of the assumptions. If there are interactions between the supply of one good and the price of another, these will manifest themselves through the existence of more conserved extensive quantities, and taking these into account will yield a more realistic model. Another weakness of the model arises in relation to the Law of One Price, which tells us that two economies are in equilibrium with each other when the price of each commodity is the same in both economies. Both cases of our model predict that, for each good Y,

$$p_Y^{(1)} = p_Y^{(2)} \quad \text{iff} \quad \frac{Y^{(1)}}{M^{(1)}} = \frac{Y^{(2)}}{M^{(2)}},$$

so that in equilibrium the goods will be shared equitably between both economies. This too is unrealistic, for if two countries of different wealth engage in trade, we expect that the trade balance be maintained to a large extent, if not widened. This strongly suggests that there is an important conserved extensive quantity which we have neglected in our analysis: we will address this issue in forthcoming work.

Conclusion

The term entropy is familiar to economists, most often encountered in the context of information theory. Here entropy refers to a quantitative measure of the intuitive notion of the randomness of a probability distribution which was formulated by Shannon & Weaver (1969). It is used in a statistical procedure to obtain an unbiased estimate of a probability distribution. Given conditions that the distribution must satisfy, the least biased estimate is that for which the entropy is a maximum. Any other estimate contains more information, and amounts to making unwarranted assumptions about the distribution. While this entropy arises from a large deviation principle, such as we have introduced, there are two significant differences in its interpretations:

1. We have used entropy directly as an equilibrium concept. The interpretation given to entropy in information theory ignores the large deviation principle from which it arises, and hence any notion of equilibrium.

2. In information theory no importance is attached to the numerical value of the entropy — it is only used as a statistical tool for choosing unbiased estimates of probability distributions. In contrast, entropy has real significance in physics, where it quantifies our experience of heat, and in economics, where it is a measure of utility.

One striking feature of thermodynamics is that it gives very general predictions about matter without making any assumptions about its microscopic nature. The thermodynamic formalism provides a general conceptual framework in which specific hypotheses about equilibrium can be formulated and examined: our model is one such hypothesis, and the framework highlights its inadequacies and suggests remedies. We present it is order to make the methodology of section I operational. As a minimal test of adequacy the model corroborates well-established postulates (such as the inverse relationship between prices and quantity). What is of greater interest is the physical analogies which

10

can be drawn with general equilibrium, yielding fresh insights into economics phenomena and producing novel interpretations of familiar concepts. The implication is that economists may have much to learn from physicists and that physics may help economists to explore and understand topics in equilibrium economics.

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