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ON QUARK CONFINEMENT

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Abstract A sufficient condition for the confinement of quarks is presented. Quarks are shown to be unobservable. Colour singlets are however, observables. The results of deep inelastic scattering are discussed. We argue that QCD does not exhibit a deconfining transition.

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Quantum Chromodynamics (QCD) is the theory of strong interactions^[1]. Its success is based on perturbation theory. The content of the theory is a non-abelian, $SU(3)$, interaction of quarks and gluons. Evidence for these particles comes from deep inelastic scattering. The outstanding problem in QCD is that these particles have not been observed experimentally. This has led to the confinement hypothesis that only colour singlet objects are observed in nature. In this letter we will prove that this is indeed the case.

The QCD Lagrangian is

$$\mathcal{L} = -\frac{1}{4}F^2 + \bar{\psi}(i\not{D} - m)\psi, \quad (1)$$

where F is the field strength constructed out of the non-abelian potentials A , D is the covariant derivative and ψ is a fermionic field. This Lagrangian exhibits the following gauge invariance

$$\begin{aligned} A(x) &\rightarrow A^g(x) = g^{-1}(x)A(x)g(x) + g^{-1}(x)dg(x) \\ \psi(x) &\rightarrow \psi^g(x) = g^{-1}(x)\psi(x). \end{aligned} \quad (2)$$

Observables must be gauge invariant. From Eq. 2 we see that the fermionic fields, ψ and $\bar{\psi}$, are not observables and thus cannot be identified with observable quarks. A similar problem for the electron arises in QED and has been solved by Dirac^[2] as we now explain (see also Ref. 3).

The physical electron field is given by

$$\psi_{\text{phys}}(x) = \exp\left(ig\frac{\partial_i A_i(x)}{\nabla^2}\right)\psi(x). \quad (3)$$

From the abelian version of (2) this may be straightforwardly seen to be gauge invariant (or, more properly, BRST invariant^[3]) up to rigid (global) transformations. This physical field is actually the electron: its propagator is gauge invariant^[4]. In contrast to the usual asymptotic identification of the electron with ψ , this electron has an electromagnetic charge and creates a Coulomb electric field^[2].

An alternative approach to this is as follows. Consider a fermion attached to a Wilson line

$$\psi_{\Gamma}(x) = \exp\left(ig\int_{-\infty}^x A_{\mu}(z)dz^{\mu}\right)\psi(x), \quad (4)$$

where Γ is any contour from the point x to $-\infty$. Although this is, by construction, gauge invariant, it is dependent on the arbitrary line Γ . A physical electron *cannot* have this property. Exploiting the gauge invariance of the theory, we set the unphysical field A_0 to zero. The spatial components may be decomposed into the physical, transverse components, A_i^T , and the unphysical, longitudinal component, $A_i^L = \frac{\partial_i \partial_j A_j}{\nabla^2}$. This means that ψ_Γ may be written as

$$\psi_\Gamma(x) = N_\Gamma(x) \psi_{\text{phys}}(x), \quad (5)$$

where

$$N_\Gamma(x) = \exp \left(ig \int_{-\infty}^x A_i^T(z) dz^i \right). \quad (6)$$

This gauge invariant normalisation factor contains all the contour dependence and must be removed for the fermion to have any physical meaning. We thus recover Dirac's physical electron.

A sufficient condition for the confinement of quarks would be to show that no contour and gauge independent generalisation of Dirac's physical electron can be constructed for the quarks. We will now demonstrate that this is the case.

Working in a Hamiltonian description^[5] of QCD, where the momenta conjugate to the potential is denoted by $\Pi(x)$, i.e., such that the fundamental Poisson bracket is $\{A_i^a(x), \Pi_b^j(y)\} = \delta_b^a \delta_i^j \delta(x - y)$, we see that if such a field exists it may be written as

$$\psi_{\text{phys}}(x) = V(x) \psi(x), \quad (7)$$

where $V(x)$ is a field dependent element of $SU(3)$. This implies that under a gauge transformation V must transform as

$$V(x) \rightarrow V^g(x) = V(x)g(x). \quad (8)$$

We now assume that for the fundamental fermions of the system this V may be taken as a function of the gauge fields only. Thus writing

$$V(x) = \exp(iv(x)^a T^a), \quad (9)$$

where T^a denotes the Gell-Mann matrices, the infinitesimal version of (8) is

$$\{v^b(x), G^a(y)\} = \delta^{ab} \delta(x - y), \quad (10)$$

where G^a is the infinitesimal generator of gauge transformations

$$G^a(x) = (D_i \Pi^i)^a(x) + g J_0(x), \quad (11)$$

and J_0 is the current density. If such a V existed, then Eq. 10 would imply that $v^b(x)$ is a gauge fixing condition. We now assume that our fields are chosen so that, as far as the gauge group is concerned, we can identify the space time with $\mathbb{R} \times S^3$, where S^3 is the spatial three sphere. As is well known^[6] there is no such global gauge fixing in QCD (the Gribov ambiguity). Hence there is no gauge invariant description of a single quark. Of course there are observables in QCD, these correspond to gauge invariant combinations of the fundamental fields; an example is $\bar{\psi}\psi$.

We stress that the above is a *sufficient* condition for confinement, and is not a necessary one. Indeed, abelian theories (for example, compact U(1) in three dimensions^[7]) may also confine due to dynamical effects. We now discuss some further consequences of this proof of confinement.

Locally, that is at small scales or high energies, gauge fixings of the form (10) can be constructed. Thus at such scales quarks will appear to be physical. Therefore they can be ‘observed’ in deep inelastic scattering. In such a local description string like singularities could be expected. To find the scale of confinement is, however, a hard dynamical question.

In order to describe QCD at finite temperature and density our assumptions on the topology of space time must be replaced by $S^1 \times S^3$. This additional complication of the topology will not alter our arguments, thus we predict that there is no deconfining transition, although the scales will change.

Another kinematical account of confinement has been proposed by Kugo and Ojima^[8]. The connection between their work and ours is unclear to us, in particular they make no reference to the role of the Gribov ambiguity.

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