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HEATING FIELD THEORY THE †
"ENVIRONMENTALLY FRIENDLY" WAY!

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ABSTRACT

We discuss how to implement an "environmentally friendly" renormalization in the context of finite temperature field theory. Environmentally friendly renormalization provides a method for interpolating between the different effective field theories which characterize different asymptotic regimes. We give explicit two loop Padé resummed results for $\lambda\phi^4$ theory for $T > T_c$. We examine the implications for non-Abelian gauge theories.

1. Introduction

Many of you will no doubt, at some time or other, have tried to understand the infrared structure of finite temperature field theory. There are many techniques available: "daisy" resummations, two particle irreducible effective action, $1/N$ expansions, ε expansions etc.. We suspect however that you haven't been completely satisfied with any of them. In this paper we will discuss a methodology which offers significant advantages over other techniques — "environmentally friendly" renormalization. The approach was initiated by O'Connor and Stephens[1], and further developed in subsequent papers[2]. A full and recent account[3] of "environmentally friendly" methods is available, here, because of space constraints, we can only offer a brief summary Let us first remind ourselves about some of the main problems of finite temperature field theory. When one heats a theory up one is basically using the temperature parameter as a means of changing "scale". Physical systems generically exhibit effective degrees of freedom (EDOF) which are radically different at different scales. The keyboard at which one of us is writing this paper for instance has 86 keys which can be depressed or undepressed giving rise to 86 EDOF. Obviously at scales $\sim 10^{-6}$cm the EDOF are quite different. It would be quite inappropriate to describe the typing process in terms of the atoms which make up the

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keys. Equally, a description of microscopic physics in terms of typewriter keys is inappropriate. Additionally if I raised the temperature of the typewriter eventually the natural degrees of freedom would be gas molecules. The example might seems somewhat facetious but it has a serious point. In QCD or the electroweak theory the EDOF at temperatures $\sim 1\text{Mev}$ are very different to those at temperatures $\sim 300\text{Mev}$. More prosaically $\lambda\phi^4$ theory, a paradigm for the Higgs sector of the standard model, exhibits very different degrees of freedom at very high and very low temperatures.

The basic problem of finite temperature field theory is to quantitatively describe systems which exhibit very different degrees of freedom at different "scales" — temperatures. In the next section we will outline the basis of environmentally friendly renormalization wherein such systems can be examined. In section 3 we will discuss exactly what one means by a high temperature system. In section 4 we will present results for $\lambda\phi^4$ theory and in section 5 discuss gauge theories. Finally in section 6 we'll draw some conclusions.

2. "Environmentally Friendly" Renormalization

Field theories describe systems that have many degrees of freedom. In such systems fluctuations inevitably play a very important role. Renormalization judiciously applied provides a method of capturing the effects of fluctuations in the parameters of the theory. The bare parameters with which one begins a field theory calculation are suitable for describing systems in the absence of fluctuations and describe in a meaningful way the EDOF of the system under such circumstances. Fluctuations change the nature of the EDOF to such an extent that the bare parameters become totally unsuited to describing them. One gets round this problem by using the freedom to reparametrize the system, transforming to a set of parameters, the renormalized ones, that offer a more faithful representation of the EDOF.

Particle physicists have traditionally seen the UV divergences seemingly inherent in a continuum description as the reason for renormalization. There is nothing inherent in the theory of renormalization itself to warrant this. Large fluctuation effects can in priciple originate anywhere in the spectrum. The canonical words: "divergences", "strong-coupling" and "non-perturbative" are almost the inevitable consequence of trying to describe a system with strong fluctuations in terms of an inappropriate set of parameters. What one requires is a renormalization/reparametrization to a set of parameters suitable for describing the system at the scales of interest.

In the case of finite temperature field theory the EDOF are temperature dependent. It therefore seems sensible to implement a renormalization which is temperature dependent in order that one might track the evolving nature of the EDOF as a function of temperature. The imposition of a temperature independent renormalization yields a very badly behaved perturbative series. In the high temperature limit perturbation theory breaks down and (IR) divergences appear. This breakdown is telling us that the renormalized parameters being used are totally inadequate for describing the high temperature limit, being those appropriate for $T = 0$, e.g. $T = 0$

masses and coupling constants. If one thinks of renormalization as a coarse graining procedure, then when one changes renormalization scale, i.e. implements an RG transformation, using $T=0$ renormalized parameters, one is effectively coarse graining $T=0$ EDOF. As one heats the theory up the disparity between the "true" EDOF of the system and those effectively being coarse grained becomes greater, hence the breakdown in perturbation theory.

All the above undesireable effects can be avoided by implementing a more suitable RG. We will show this more explicitly in the next section. Besides temperature there are many other parameters that could induce changes in the EDOF: e.g. electric/magnetic fields, spacetime geometry, anisotropic interactions etc. We dub such parameters "environmental" as we are generically trying to describe fluctuations in a field theory in an "environment" described by these parameters. Thus in QED changing a background magnetic field changes the environment in which the electrons and photons see themselves. As the EDOF are sensitive to the environment, we will call an RG which tracks the changing nature of them as the environment changes "environmentally friendly"

3. The Environmentally Friendly Way

In this section we will consider finite temperature $\lambda\phi^4$ theory. We will be sketchy leaving the reader to get more details from the papers[1,2,3]. We determine our renormalized parameters via the normalization conditions

$$\Gamma^{(2)}(k=0, m^2=\kappa^2, \lambda, T, \kappa) = \kappa^2 \tag{1}$$

$$\frac{\partial\Gamma^{(2)}}{\partial k^2}(k, m^2=\kappa^2, \lambda, T, \kappa)|_{k=0} = 1 \tag{2}$$

$$\Gamma^{(4)}(k=0, m^2=\kappa^2, \lambda, T, \kappa) = \lambda \tag{3}$$

$$\Gamma^{(2,1)}(k=0, m^2=\kappa^2, \lambda, T, \kappa) = 1 \tag{4}$$

κ being an arbitrary renormalization scale. Using these conditions, and implementing a [2,1] Padé resummation of the two loop Wilson functions we obtain

$$\beta(h,\tau) = -\varepsilon(\tau)h + \frac{h^2}{1+4\left(\frac{(5N+22)}{(N+8)^2}f_1(\tau) - \frac{(N+2)}{(N+8)^2}f_2(\tau)\right)h} \tag{5}$$

$$\gamma_{\phi^2}(h,\tau) = \frac{(N+2)}{(N+8)}\frac{h}{1+6\frac{1}{(N+8)}(f_1(\tau)-\frac{1}{3}f_2(\tau))h} \tag{6}$$

and

$$\gamma_\phi(h,\tau) = 2\frac{(N+2)}{(N+8)^2}f_2(\tau)h^2 \tag{7}$$

where $\varepsilon(\tau) = 1 + \tau\frac{d}{d\tau}\ln(\sum_n m^{-3})$, $\tau = \frac{T}{\kappa}$

$$f_1(\tau) = 2\frac{\sum_{n_1,n_2}\left(\frac{1}{m_1^3}\left(\frac{1}{M}-\frac{1}{2m_2}\right)+\frac{1}{m_1M^2}\left(\frac{1}{m_1}+\frac{2}{m_2}\right)\right)}{\left(\sum_n\frac{1}{m^3}\right)^2} \quad \text{and} \quad f_2(\tau) = 4\frac{\sum_{n_1,n_2}\frac{1}{M^3m_1}}{\left(\sum_n\frac{1}{m^3}\right)^2}$$

with $m_i = (1 + \frac{4\pi^2 n_i^2}{\tau^2})^{\frac{1}{2}}$, $m_{12} = (1 + \frac{4\pi^2}{\tau^2}(n_1 + n_2)^2)^{\frac{1}{2}}$, $M = m_1 + m_2 + m_{12}$. In Eq.'s (5-7) the coupling h, or floating coupling[1], is defined via the relation $h = a_2(\frac{T}{\kappa})\lambda(\kappa)$, where a_2 is the coefficient of λ^2 in $\beta(\lambda)$. Eq.'s (1) and (2) imply that κ is the inverse finite temperature screening length $m(T)$, thus the Wilson functions depend on $\frac{T}{m(T)}$ and h.

Eq. (5) exhibits more than one fixed point. As $\frac{T}{m(T)} \to 0$, one obtains the Gaussian fixed point as expected in four dimensions. As $T \to T_c$, i.e. $m(T) \to 0$ one finds a non-trivial fixed point, $h = 1.732$ for $N = 1$, for instance. The value of the fixed point and the corresponding critical exponents are in exact agreement with corresponding two loop Padé resummed results[4] in three dimensional critical phenomena. Of course we could have assumed that our theory was three dimensional near the phase transition but why do that when we can derive it. We can see the complete crossover between four and three dimensional behaviour as $\frac{T}{m(T)}$ varies between 0 and ∞ in a perturbatively controllable fashion. One can think of $d_{eff} = 4 - \varepsilon(\frac{T}{m(T)})$, which interpolates between 4 and 3 when $\frac{T}{m(T)}$ varies between 0 and ∞, as a measure of the effective dimension of the system. Near $T = 0$ the EDOF of the problem are 4 dimensional and near $T = T_c$, 3 dimensional. Between these two extremes they are neither 4 nor 3 dimensional. The power of our approach is that we have implemented a reparametrization which is temperature dependent in such a fashion that it tracks the evolving nature of the EDOF between the 4 and 3 dimensional limits. In the figure we present a plot of $\gamma_\phi(\frac{T}{k})$ versus $\ln\frac{T}{k}$, where the solution of $\kappa\frac{d\lambda}{d\kappa} = \beta$ has been used. Its physical significance derives from the fact that at $T = T_c$,

$$G^{(2)}(k,T) \sim \frac{e^{\int^k \gamma_\phi}}{k^2} \tag{8}$$

where k is the spatial momentum.

The parameter with which we are investigating the crossover here is the finite temperature screening length. Though it is a very natural parameter in the real world often we only have access to the zero temperature parameters. One would therefore like to know how to describe the crossover in terms of them. Here we performed a renormalization of the system at the physical temperature T and used the screening length as a running scale. We could have renormalized at a completely arbitrary value of the temperature instead[5]. In this case we would be running the environment itself. By so doing however one can relate finite temperature quantities to zero temperature quantities quite easily. More will be said about such schemes in another article in these Proceedings.

4. "Watching the daisies grow"

One of the most oft used methods of treating IR problems in finite temperature field theory, and certainly in $\lambda\phi^4$ theory, is to resum the daisy diagrams on the basis that they are the most IR divergent as temperature increases. It is important to have an intuitive understanding of the approximation procedure one is implementing. We have motivated our methods in terms of a reparametrization which tracks the EDOF of the system as the environment changes. If perturbation theory works you can be fairly sure you are tracking the EDOF well. If it breaks down the opposite is true.

When one starts with $\lambda\phi^4$ theory at $T = 0$, with a zero temperature mass m, one finds that when $T \gg m$ ($T > T_c$) perturbation theory breaks down due to the presence of a large thermally induced mass $m(T) = \lambda^{\frac{1}{2}}T$. Perturbation theory is breaking down because one is trying to use small mass EDOF to describe a system where the true EDOF are very massive relative to the scale m. By resumming the daisies one is effectively expanding around a theory of mass $m(T)$ as opposed to a mass m. Once this is done one finds that perturbation theory works. So, obviously something was done correctly. But what is being described? Certainly not the vicinity of a second order or weakly first order phase transition where $m(T) \ll T$. Daisy resummation methods alone cannot be used to describe such a regime because the physical characteristic of the regime, $m(T) \ll T$, is totally different to that of the daisy resummation, $m \ll T$. When one starts at $T = 0$ with $\bar{\phi} = 0$ and heats the theory up, then one drives the theory away from a phase transition not towards it. What regime is far away from the critical regime in critical phenomena? The mean field regime, and it is precisely this regime which the daisy resummation is suitable for. Once the large mass shift has been accounted for, one is in a regime where IR fluctuations are strongly suppressed by the effects of the large mass — hence loop corrections become unimportant. This is the one regime where it is not necessary to use RG methods (though certainly RG methods can also describe this regime) because of the fact that fluctuations are unimportant.

4. Gauge Theories

We have tried to point out so far the intuitive basis of our methodology and used $\lambda\phi^4$ theory as an interesting test ground for it (besides the Higgs sector it can also successfully describe in one guise or another a myriad of physical systems in statistical physics[3]). We are interested in investigating how the EDOF of a physical system are sensitive to the environment they "feel" themselves to be in. If we consider non-trivial background fields then they too form part of the environment in which the fluctuations of the system exist. These background fields need not be homogeneous but could represent such interesting objects as instantons, solitons, monopoles, vortices etc. As is well known such non-trivial solutions of the classical field equations play a very important role in gauge theories. Hence we would expect the environment in theories with gauge fields to be very rich and complex. Gauge theories are therefore archetypal crossover problems — in QCD the low energy EDOF are entirely different from the high energy ones. The growth of the QCD coupling in the IR is symptomatic of the fact that the EDOF are no longer quarks and gluons but hadrons and mesons. In principle this is no different than the breakdown of perturbation theory in the IR limit of finite temperature $\lambda\phi^4$ theory. The crucial difference is that in the latter we know exactly how to describe the effects of temperature as an environmental parameter. We know much less about how to describe the environmental effects of the QCD vacuum.

So what can we say about finite temperature gauge fields? The first important fact is that there is a strong anisotropy in gauge theories as to the effects of temperature. The electric and magnetic sectors react in very different ways. In abelian gauge theories the electric sector acquires a thermal mass $\sim eT$, while the magnetic sector remains massless. The electric sector is then very akin to $\lambda\phi^4$ theory for $T \gg m$. Fluctuations acquire a large mass and essentially become unimportant leading to mean field like behaviour. The magnetic sector remains massless throughout and one might be tempted to think that it is purely described by a three dimensional gauge field. However, there is an interesting crossover between three and four dimensional massless behaviour as a function of $\frac{T}{k}$ where k is the typical momentum scale of the magnetic process under study. In all these cases an environmentally friendly RG can proffer a reliable description. That is not to say that there are no subtleties to beware of. In scalar electrodynamics for instance vortices can play an important role in lower dimensions, and one would consequently expect to see a breakdown in renormalized perturbation theory if this fact is ignored. The latter indicates that the original model for the environment (i.e. ignoring vortices) was inadequate.

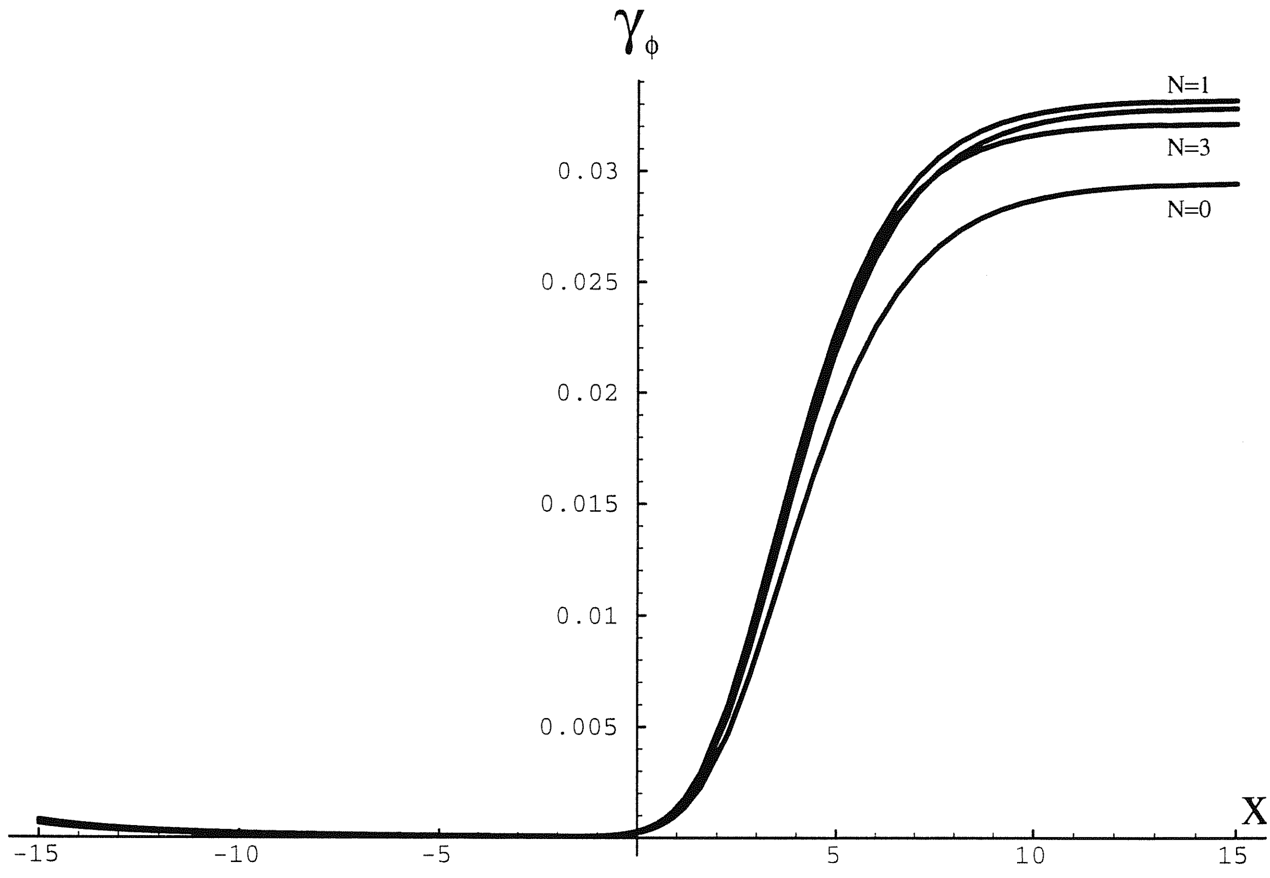
Non-Abelian gauge theories are even more complicated. As in the abelian case the electric sector acquires a thermal mass $\sim gT$ and leads to a mean field regime. The magnetic sector however also picks up a thermal mass, but only at the two loop level. If one works to one loop in RG improved perturbation theory, where the only environmental variable accounted for is temperature, one finds that due to the absence of an IR cutoff one is driven into a strong coupling regime[6]. Thus the simple prescription that works for $\lambda\phi^4$ theory is inadequate. The moral is clear: thermally corrected quarks and gluons are not the appropriate EDOF. The effect of the temperature is to drive the theory into the confining regime and unfortunately the model of the environment we are using is not sophisticated enough to account properly for the effects of the QCD vacuum. One might argue that including two loop effects in the thermal RG would alleviate this problem. This seems unlikely. Not only must a magnetic mass be introduced but it must increase with temperature sufficiently fast so as to act as an efficient IR cutoff at all temperatures.

Conclusions

In this short note we have described the essence of environmentally friendly renormalization which provides a method for capturing the crossover from one effective field theory to another. Effective field theory focusses on a narrow band of energy scales within which the EDOF don't qualitatively change. The power of our method is its ability to capture the large qualitative changes in EDOF inherent in the crossover between effective field theories. The example we have concentrated on here is finite temperature field theory, and in particular $\lambda\phi^4$ theory, where for $\frac{T}{m(T)} \sim 0$ the effective field theory is four dimensional, and for $\frac{T}{m(T)} \gg 1$ the effective field theory is three dimensional. We presented two loop Padé resummed expressions for the Wilson functions which capture fully this crossover as exemplified in the figure. We also discussed briefly some of the subtleties inherent in describing finite temperature gauge fields, a subject we will return to in the future.

6. References

1. Denjoe O'Connor and C.R. Stephens, *Nucl. Phys.* **B360** (1991) 297.
2. Denjoe O'Connor, C.R. Stephens and F. Freire, *Class. Quan. Grav.* **23** (1993) S243; Denjoe O'Connor, C.R. Stephens and F. Freire, *Mod. Phys. Lett.* **A25** (1993) 1779; F. Freire and C.R. Stephens, *Zeit. Phys.* **C** (to be published).
3. Denjoe O'Connor and C.R. Stephens, "Environmentally Friendly Renormalization"; Preprint THU-93/14, DIAS-STP-93-19; (to be published in *Int. Jou. Mod. Phys.* **A**.
4. G.A. Baker, B.G. Nickel and D.I. Meiron, *Phys. Rev.* **B17** (1978) 1365.
5. H. Matsumoto, Y. Nakano and H. Umezawa, *Phys. Rev.* **D29** (1984) 1116.
6. M.A. van Eijck, C.R. Stephens and Ch.G. van Weert, "Temperature Dependence of the QCD Coupling" ; Preprint THU-93/08, ITFA-93-11.



Graph of γ_ϕ against $x = \ln(T / k)$ for $N = 0,1,2$ and 3