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Finite temperature phase transitions in quantum field theory

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Abstract. Using the formalism developed in [1] we discuss finite temperature quantum field theory in the high temperature regime $T >> m_T$ where m_T represents a generic finite temperature mass in the theory. In particular we consider $\lambda\phi^4$ theory in four dimensions showong perturbatively that it has a non-trivial fixed point at finite temperature, the relevant anomalous dimensions near the second-order phase transition being three-dimensional ones. We emphasize the importance of having renormalization schemes and a renormalization group (RG) equation that can explicitly take into account the fact that the degrees of freedom of a theory may be qualitatively different at different scales.

Finite temperature field theory (FTFT) has been a subject of interest for some time, especially in light of the importance of FT phase transitions in the early universe. One of the main problems in FTFT has been analysing the 'high' temperature limit (HTL) where $T >>$ all other length scales in the problem. Note that $T = 0$ scales being $\ll T$ does not necessarily imply that one is at high temperature. e.g. $m(T = 0) << T$ need not imply that $m_T << T$. In the HTL conventional perturbation theory (PT) breaks down. For T independent renormalization schemes, such as minimal subtraction, unless $T \sim \kappa$, where κ is the renormalization scale, PT is ill defined. If one attempts to improve things using a RG based on such schemes the resultant PT remains ill defined in the HTL. The reason for this is quite simple. Just as bare parameters provide a bad perturbative description of the theory when $\Lambda/\kappa \to \infty$, so $T = 0$ parameters provide a bad perturbative description when $T/\kappa >> 1$. In the HTL the theory has different degrees of freedom, three dimensional (3D) ones in fact, than at $T = 0$. This is not new, what is new is the RG interpretation of the breakdown of conventional FTFT and the provision of a qualitative and quantitative framework for the analysis of the HTL (see [1-3] for more details). A T independent RG dresses the parameters of the theory with $T = 0$, i.e. 4D, fluctuations. In the HTL, as mentioned, these are not the relevant degrees of freedom. If one thinks of the RG intuitively as a course graining procedure what one requires is a RG that for $T \sim 0$ course grains 4D degrees of freedom and in the HTL 3D degrees of freedom. Such a RG can be derived on the basis of T dependent normalization conditions such as, using the imaginary time formalism, for $\lambda\phi^4$

$$\Gamma^{(2)}_{00}(0, m_T, \lambda, T, \kappa) = m_T^2 \qquad \Gamma^{(4)}_{0000}(0, m_T, \lambda, T, \kappa) = \lambda T. \tag{1}$$

The consequent RG is explicitly T dependent, hence so are the β function and anomalous dimensions. Some explicit results to 2 loops are

$$\beta(h) = -\varepsilon\left(\frac{T}{m_T}\right)h + h^2 - \frac{4}{3}\left(f_1 - \frac{1}{9}f_2\right)h^3 + O\left(h^4\right) \tag{2}$$

$$\gamma_{\phi^2}(h) = \frac{h}{3} - \frac{2}{9}\left(f_1 - \frac{1}{3}f_2\right)h^2 + O\left(h^3\right) \tag{3}$$

$$\gamma_{\phi}(h) = \tfrac{2}{27} f_2 h^2 + O\left(h^3\right) \tag{4}$$

where

$$\varepsilon\left(\frac{T}{m_T}\right) = 1 - m_T \frac{d}{dm_T} \ln\left(\sum_n \left(1 + \frac{4\pi^2 n^2 T^2}{m_T^2}\right)^{-3/2}\right)$$

$$f_1\left(\frac{T}{m_T}\right) = 2\sum_{n_1,n_2} m_1^{-1}\left(m_1^{-2}\left(M^{-1} - \tfrac{1}{2}m_2^{-1}\right)\right.$$
$$\left. + M^{-2}\left(m_1^{-1} + m_2^{-1} + m_{12}^{-1}\right)\right)\left(\sum_{n_1} m_1^{-3}\right)^{-2}$$

$$f_2\left(\frac{T}{m_T}\right) = 4\sum_{n_1,n_2} M^{-3}\left(m_1^{-1}\right) \times \left(\sum_{n_1} m_1^{-3}\right)^{-2}$$

and $m_i = (1 + 4\pi^2 n_i^2 T^2/m_T^2)^{1/2}$, $m_{12} = [1 + (4\pi^2 T^2/m_T^2)(n_1 + n_2)^2]^{1/2}$, $M = (m_1 + m_2 + m_{12})$. The coupling $h = a_2(T/m_T)\lambda$, where a_2 is the coefficient of the $O(\lambda^2)$ term in $\beta(\lambda)$. These equations interpolate in a smooth fashion completely across the crossover as a function of T/m_T yielding as $T/m_T \to 0$ characteristic 4D values and as $T/m_T \to \infty$ 3D values. $\beta(h) = 0$ from (2) describes a 'floating' fixed point that captures the essence of the crossover without having to solve it as a differential equation. The effective expansion parameter is $\varepsilon(T/m)$ which varies between 0 and 1. In the HTL $\varepsilon \to 1$ and in order to obtain good quantitative accuracy one should work to multiloop order and Borel resum. With the conventional PT of FTFT; for $\lambda \sim 10^4$, $T/m \sim 10^8$ the effective expansion parameter $\sim 10^4$, whereas in our framework it is a number slightly less than 1.

In the large N limit of scalar electrodynamics one obtains a fixed point and anomalous dimensions analogous to (2-4). For QCD as long as all relevant scales κ are $\gg \Lambda_{QCD}$, then as T/κ varies between 0 and ∞ we expect to see a crossover from a 4D logarithmic approach to the Gaussian fixed point to a 3D power law approach. More will be said about gauge theories in forthcoming articles.

References

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