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ENERGY SPECTRA IN INFLATIONARY MODELS

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ABSTRACT

Curvature effects in inflationary Robertson-Walker models, although negligible at the present day, can drastically alter the energy spectra of non-zero spin quantum fields in $k \neq 0$ universes at early times. This affects the thermal energy densities of quantum fields, and hence the dynamical evolution of the universe. It is argued that, for $k \neq 0$ universes with temperature T and spatial curvature R, if the inflation factor were so large that $RT \lesssim 10^{-6+2k}$ before inflation, then there was never a pre- inflationary era in new inflation when the universe evolved according to the classical Einstein equations, but that the inflationary epoch must have grown directly out of the quantum gravity era.

Inflation offers an intriguing explanation of why the universe appears so flat at the present day. Of the three possibilities for the spatial curvature of a Robertson-Walker universe, $k = 0, \pm 1$, $k = 0$ is a set of measure zero and would require a very special set of initial conditions to be realised. But if $k \neq 0$ the observed value of the spatial curvature is much smaller than any "natural" value. Inflation cures this by reducing the spatial curvature by many orders of magnitude. In chaotic inflation[1] the inflationary era is regarded as emerging directly from the quantum gravity domain. In new[2,3] and extended[4] inflation the universe is usually regarded as undergoing a standard, radiation dominated expansion (or a modified power law for extended inflation) from the time at which it emerges from the mysterious quantum gravity era, at about $10^{-43}s$, until the time when the energy density of the scalar field starts to dominate the dynamics, often taken to be about about $10^{-35}s$, the time at which a GUT phase transition might be expected to start. In this talk it will be argued that if the inflation factor were so large that $RT \lesssim 10^{-6+2k}$ before inflation, then that there was never any such era and inflationary era must have emerged *directly* from the quantum gravity era in new and extended as well as chaotic inflationary models[5].

The basic observation is that the energy spectra of a quantum fields with spin s in a $k \neq 0$ static Robertson-Walker universe is not quite of Planckian form, but is distorted by a term of order $(RT)^{-2}$, where R is the scale factor and T is the temperature. In an open Einstein ($k = -1$) universe the energy density is given by[6] (natural units are used in which $\hbar = c = k_B = 1$, where k_B is Boltzmann's constant),

$$\begin{aligned}\rho_- &= \frac{g_s T^4}{2\pi^2}\int_0^\infty \frac{x^3+\left(\frac{s}{RT}\right)^2 x}{e^x-(-1)^{2s}}dx \\ &= \frac{g_s\pi^2 T^4}{96}\left[\left(\frac{15+(-1)^{2s}}{5}\right)+\left(\frac{s}{\pi RT}\right)^2\left(\frac{9+7(-1)^{2s}}{2}\right)\right],\end{aligned} \tag{1}$$

where g_s is the number of degrees of freedom for the field, $g_s = 2$ for photons. $s = 0$ refers only to the conformally coupled wave equation for scalars.

For a closed closed ($k = +1$) Einstein universe it is[7,8]

$$\begin{aligned}\rho_+ &= \frac{g_s}{2\pi^2 R^4}\sum_{n=1}^{\infty}\frac{n^3-s^2 n}{e^{n/(RT)}-(-1)^{2s}}+\frac{a(s)}{\pi^2 R^4} \\ &= \frac{g_s T^4}{2\pi^2}\frac{1}{RT}\sum_{n=1}^{\infty}\frac{x_n^3-\left(\frac{s}{RT}\right)^2 x_n}{e^{x_n}-(-1)^{2s}}+\frac{a(s)}{\pi^2 R^4},\end{aligned} \tag{2}$$

where $x_n = n/(RT)$. The second term here represents the Casimir energy on the three sphere[8]. Again, the $s = 0$ case is only for the conformally coupled scalar wave equation. In particular the spin dependent co-efficients are $a(0) = 1/480$, $a(1/2) = 17/1920$, $a(1) = 11/240$.

For a static spatially flat ($k = 0$) universe, there is no deviation from the usual Planck form and the considerations in this talk are irrelevant, but this will be regarded as being a very special situation requiring special initial conditions.

Of course, in the real world the universe is not static and the Robertson-Walker scale factor, $R(t)$, is not a constant so the thermal spectra are not known exactly in general, but as long as the spectrum is only analysed in the frequency range $\nu >> H$, where $H = \dot{R}/R$ is Hubble's "constant", then one would expect the spectrum to be given reasonably accurately by its static form.

For the present day cosmic microwave backround, with $T = 2.7^o K \sim (1mm)^{-1}$ and $R \gtrsim 10^{28}$cm, $RT \gtrsim 10^{29}$ and so the distortion from the Planckian spectrum is less than one part in 10^{58} and is completely negligible. Without inflation, the distortions are negligible at all earlier times. This follows simply from the adiabatic expansion of the standard Big Bang model. Since the total entropy, S, is constant we have $S = sR^3 \sim T^3R^3 =$ constant, where $s \sim T^3$ is the entropy density. Thus as long as the total entropy remains constant, TR remains constant and the distortions are always negligible.

However if the total entropy of the universe at an earlier epoch was ever smaller than it is today, $1/(RT)$ would have been larger and the distortions from the Planck spectrum correspondingly larger. This is exactly what inflation does. It is designed to hold T fixed while R increases by at *least* twenty nine orders of magnitude, thus since $RT \gtrsim 10^{29}$ after inflation, we might expect to have $RT \lesssim 1$

before inflation, and the distortion from a Planckian spectrum for fields with non-zero spin is significant, and even dominant if $RT << 1$.

The shape of the spectra for photons is very different from the Planckian form if $RT << 1$. For $k = -1$ the energy density per unit frequency is non-zero at zero frequency (specifically, it is $T^4/(\pi RT)^2$) and there is much more energy density at low frequencies than in the Planck spectrum, the distortion being larger for smaller values of RT. For values of RT less than 0.619 there is no longer a peak and the spectrum is monotonically decreasing.

For $k = +1$ the thermal spectrum is discrete and there is less energy at low frequencies than in the Planck spectrum. For $RT << 1$, the thermal spectrum is strongly supressed and almost all the energy is in the Casimir term.

For values of $RT << 1$ the thermal history of the universe before inflation is also modified. For the Planck spectrum $\rho_0 = \frac{\pi^2}{30} N_{eff} T^4$ where $N_{eff} = N_b + \frac{7}{8} N_f$ is the effective number of degrees of freedom (1 per bosonic degree of freedom and 7/8 per fermionic degree of freedom). But for $k \neq 0$, $\rho_\pm >> \rho_0$ when $RT << 1$ (by a factor of order $(RT)^{-2}$ for $k = -1$ and $(RT)^{-4}$ for $k = +1$ - in the latter case, the Casimir energy is the dominant contribution).

For example in new inflationary models during the pre-inflation, radiation dominated era the age of the universe goes like $t \propto 1/\sqrt{\rho}$. Thus, for a given temperature, a $k \neq 0$ universe is much younger than a $k = 0$ one would be (by a factor of order $(RT)^{-1}$ for $k = -1$ and $(RT)^{-2}$ for $k = +1$). If the inflation factor in a $k = +1$ universe is so large that $RT \lesssim 10^{-4}$ before inflation then the universe is younger by a factor $\gtrsim 10^8$ than is usually assumed. So instead of setting in at $10^{-35} s$ inflation would have started at $10^{-43} s$, i.e. directly from the quantum gravity era! For a $k = -1$ universe a pre-inflation value of $RT \lesssim 10^{-8}$ gives the same conclusion.

We have seen that, if the inflation factor is large enough to give a pre-inflation value of $RT \lesssim 10^{-6+2k}$, then there was never a pre-inflationary era in new inflation during which the universe evolved classically. Such inflationary factors are not at all uncommon in these models. Thus, in contrast to the usual assumption, the thermal history of the very early, pre-inflation, universe is very sensitive to the value of k.

For models of extended inflation, there are also power law solutions of the Einstein equations in the pre-inflationary era, with $t \sim \rho^{-\alpha}$ where α is model dependent. For these models similar conclusions can be drawn.

Of course, as stated earlier, the thermal spectra quoted in Eqs. 1 and 2 are only for *static* Robertson-Walker space times and are not to be trusted for frequencies smaller than or of the same order as the Hubble constant. Nevertheless, one can still expect drastic modifications of the Planck spectrum for large inflation factors.

To summarise it has been argued that, while distortions of thermal spectra for fields of non-zero spin due to curvature effects in $k \neq 0$ universes are negligible at the present day, they would have been very significant in any theory with a large production of entropy $\gtrsim 10^{87}$ at any time later than a few Planck times. In particular for new or extended inflationary models these effects would be expected to be important before the era of inflation. It would appear that the assumption of an essentially classical post-quantum gravity but pre-inflation era is not tenable.

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