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DISCRETE SYMMETRIES AND SELECTION RULES IN UNIFIED SU(8) FOR SUPERCONDUCTIVITY AND DENSITY WAVES

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ABSTRACT

We show that a discrete T symmetry produces selection rules which require certain order parameters to vanish.

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ABSTRACT

We show that a discrete T symmetry produces selection rules which require certain order parameters to vanish.

The Lie Algebra SU(8) is the dynamical algebra, or spectrum generating algebra, for the general mean field Hamiltonian which describes an interacting many-electron system capable of condensing into coexisting superconductivity and/or charge and/or spin-density wave states.(1) The mean-field Hamiltonian can be taken as $H = H_{KE} + H_{SS} + H_{DW}$ where the terms include single-particle ("kinetic") energy, singlet superconductivity, and density wave terms. These are composed of bilinear products of fermion operators:

$$H_{KE} = \sum_{k} \varepsilon_k \, a^{+}_{k\alpha} \, a_{k\alpha}$$

$$H_{SS} = \sum \Delta^* a_{k\uparrow} a_{-k\downarrow} + h.c$$

$$H_{DW} = \sum \gamma_\mu a^+_{k+Q\alpha} \sigma_\mu a_{k\alpha} + h.c.$$

The parameters are: single particle energy ε_k, measured from the Fermi energy, complex gap parameter Δ, complex CDW and SDW parameters γ_μ, where μ=o (CDW), μ = 1,2,3(SDW); $\sigma_\mu = \tau_\mu/2$ where τ_μ are the Pauli matrices (τ_o = I), and $\alpha = \uparrow$ or $\downarrow$. The "external" vector Q is prescribed. As shown elsewhere,[1] the closure under Lie bracket of all the pair operators in H generates SU(8). In another contribution to this Colloquium[2] we exhibit and discuss certain of the chains of subalgebras of the general SU(8), and the corresponding models.

In the present paper we point out the existence of discrete symmetry operations, analogous to the P,C,T symmetries of field theory. We show how a discrete symmetry leads to selection rules – in particular the vanishing of certain order parameters in a (ground) coherent state. To be specific this will be illustrated on a very useful and simple model with SO3xSO3 dynamical symmetry.

Let $\hat{Q}$ be an order operator (one of the generators of the Lie

Algebra); the corresponding order parameter in an eigenstate $|\phi\rangle$ is $\eta_Q = \langle\phi|\hat{Q}|\phi\rangle$, where $H|\phi\rangle = \lambda|\phi\rangle$. The eigenstates of the Hamiltonian are obtained from a reference state $|f\rangle$ by a rotation. The reference state $|f\rangle$ can be the filled "Fermi sea" $|f\rangle = |1,1,\ldots n_{KF} = 1,0,0\ldots\rangle$; states such as $|f'\rangle = |1,1,\ldots n_k = 1, n_{-k} = 0, 1, 1,\ldots 000\rangle$ can also be employed. Then $|\phi\rangle = R^{-1}|f\rangle$ is taken as our ground coherent state. Here the rotation R brings H to the Cartan diagonal form: $RHR^{-1} = \lambda_j h_j$ where h_j is one of the generators of the Cartan (Abelian) subalgebra of H.

Consider the simplest case for discrete symmetry. Let S be a discrete symmetry operator, which commutes with the rotation: SR=RS, and let $S|f\rangle = |f\rangle$. Then if $\hat{Q}$ has negative "S-parity", $\eta_Q = 0$. The proof is simple. Suppose that under the rotation which diagonalizes H, the operator transforms as: $R\hat{Q}R^{-1} = \xi_j \hat{h}_j + \xi_{\pm\alpha} \hat{e}_{\pm\alpha}$ where $(\hat{h}_j, \hat{e}_{\pm\alpha})$ generate the dynamical algebra and the $(\xi_j, \xi_{\pm\alpha})$ are constants. Then

$$\eta_Q = \langle\phi|\hat{Q}|\phi\rangle = \langle f|R\hat{Q}R^{-1}|f\rangle = \xi_j \langle f|h_j|f\rangle$$

since $\langle f|e_{\pm\alpha}|f\rangle=0$. But also

$$\eta_Q = \langle f|S^+SR\hat{Q}R^{-1}S^+S|f\rangle = \langle f|SR\hat{Q}R^{-1}S^+|f\rangle$$

$$= \langle f|RS\hat{Q}S^+R^{-1}|f\rangle = \langle\phi|S\hat{Q}S^+|\phi\rangle.$$

So if $S\hat{Q}S^+ = -\hat{Q}$ then $\eta_Q = 0$

We illustrate the T selection rule on a model for coexistence of superconductivity and charge density waves with dynamical symmetry SO3xSO3. It is a simplified sub-model of the SU(8) unified model.[1]. The Hamiltonian is

$$H = (\varepsilon - \varepsilon')L_3 - \gamma L_2 + \Delta K_1$$

where index k is suppressed everywhere, L_3 is the kinetic energy operator, L_2 is the CDW operator and K_1 is the (singlet) superconductivity operator. Using the "triple Nambu" notation of our work[1] on the unified SU8 model the operators in H can be written:

$$\vec{L} = (S_1 \times \tau_O, S_2 \times \tau_3, S_3 \times \tau_3)$$

$$\vec{K} = (E_O \times \tau_1, W_O \times \tau_2, - W_2 \times \tau_2)$$

The commutation rules for these operators, which verify the SO3xSO3 dynamical algebra are:

$$[L_i, L_j] = i\, \varepsilon_{ijk}\, L_k; \; [L_i, K_j] = i\, \varepsilon_{ijk}\, K_k,$$

and

$$[K_i, K_j] = i\, \varepsilon_{ijk}\, L_k.$$

Defining $\vec{J}^{(1)} = 1/2\, (\vec{L} + \vec{K})$ and $\vec{J}^{(2)} = 1/2\, (\vec{L} - \vec{K})$ then $[J_i^\alpha, J_j^\beta] = i\, \varepsilon_{ijk}\, J_k^\alpha\, \delta_{\alpha\beta}$ with $\alpha, \beta = 1,2$.

Consider the discrete time-reversal operator T: $Ta_{k\uparrow} = a_{-k\downarrow}$; and $Ta_{-k\uparrow} = -a_{k\downarrow}$. The Hamiltonian H is T-invariant, so $T^{-1}HT = H$. The rotation $R = R^{(1)} R^{(2)}$ will bring H to the Cartan form: $RHR^{-1} = E(J_3^{(1)} + J^{(2)}$. Here $E = [(\varepsilon - \varepsilon^1)^2 + \Delta^2 + \gamma^2]^{1/2}$ and $R^{(\alpha)} = \exp i [\theta_2^{(\alpha)} J_2^{(\alpha)} + \theta_1^{(\alpha)} J_1^{(\alpha)}]$ with $\alpha = 1,2$. The angles $\theta_1^{(\alpha)}$ can be obtained straightforwardly. $TRT^{-1} = R$, and for the eigenfunctions: $|\phi\rangle = R^{-1}|f\rangle$, where $|f\rangle$ is the filled (non-interacting) Fermi sea; we find $T|f\rangle = |f\rangle$ so $T|\phi\rangle = |\phi\rangle$. This model has 6 order operators $\hat{Q}_A$. From the discussion previously given, if $T\hat{Q}_A T^{-1} = -\hat{Q}_A$ then $\eta_A = \langle\phi|\hat{Q}_A|\phi\rangle = 0$. We calculated all the expectation values in this model, and we find the following results (Table I).

TABLE I

$\hat{Q}$	Type	T-Parity	$\eta_A = \langle\phi\|\hat{Q}_A\|\phi\rangle$
L_1	CDW^-	-1	0
L_2	CDW^+	+1	γ/E
L_3	KE	+1	$(\varepsilon-\varepsilon^1)/E$
K_1	SSC^+	+1	Δ/E
K_2	SSC^-	-1	0
K_3	"AS"$^-$	-1	0

Clearly all odd T-parity operators give zero value of the corresponding parameter in ground state $|\phi\rangle$ for this SC-CDW SO3xSO3 model.

The general unified SU(8) model and each of the submodels we listed in reference 1 possess a number of discrete symmetries, which produce selection rules giving vanishing order parameters. Elsewhere we shall present results of a study of these symmetries and rules.

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References

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2. Allan I. Solomon and Joseph L. Birman, "Dynamical SU(8), A Laboratory for Phase Coexistence." These Proceedings.