DIAS-STP-87-58

Gravitation and the Unification of the Fundamental Forces

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It is with great pleasure that I give this address in celebration of Professor Wayman's 60th. birthday and it is an honour for me to be invited to do so. It is also with great pleasure that I give it at Dunsink, where I and my colleagues have so often been the recipients of the Waymans' hospitality. Only ten years ago it would have been difficult for particle physicists to find common ground with astronomers, but happily this is no longer the case, and I hope that my talk will help to explain the convergence between the two fields that has begun to take place during the past decade.

One of the greatest achievements of seventeenth-century physics was the unification of terrestial and astronomical physics, that is, the observation that it is the same force, namely gravitation, that causes things to fall and the planets to revolve around the sun. This unification is now so much taken for granted that when the first Sputnik was launched it was hailed for many reasons, but not for the reason that it was actually the first direct experimental test of this unification!

However, despite this great initial unification of astronomy and terrestial physics, the two branches of physics tended to go their own separate ways in the course of the succeeding centuries and it is only in our own time that their paths have begun to converge again. Indeed, until about ten years ago, such fundamental discoveries as electromagnetism and nuclear physics were important for astrophysics only to the extent that they provided new techniques for observation and new information about the composition of matter in the stars and galaxies. One of the main reasons for the separation of astrophysics from the other branches of physics was the failure to unify gravitation with any of the other fundamental forces. Indeed, until the nineteen-seventies, the only further unification of fundamental forces that had been achieved was the late-nineteenth-century unification of electricity and magnetism, consolidated by the special theory of relativity. Attempts to carry the unification process further, in particular attempts to unify electromagnetism and gravitation, were singularly unsuccessful.

In recent times, however, the situation has begun to change. First has come the realization that the nuclear interactions are at least of the same general form (the so-called gauge, or vector-meson form) as electromagnetism[(1)], a realization that was dramatically confirmed by the 1983 experimental production of the vector mesons in question, namely the W and Z particles predicted by the simplest consistent electroweak model. Although the common form of the nuclear and electromagnetic interactions does not constitute a full unification it does constitute an important step toward unification and has also pointed toward a unification with gravity. Two questions that might then be asked are: How did modern physicists have some success in the unification of electromagnetism and nuclear forces and why have they such hopes for gravitation, when the efforts in these directions of some of the most eminent physicists, such as Einstein and Schrödinger, were so singularly unsuccessful? (An apocryphal story, which might illustrate how unsuccessful they were, is the following: Sometime in the early fifties Schrödinger submitted to the Royal Irish Academy one Friday afternoon a paper entitled 'The Unified Theory of Matter'. But on Monday morning the Editor of the Proceedings found a note from him on his desk saying 'Re the paper submitted on Friday. I have been thinking about it over the weekend and should like to amend the title to 'The Unified Theory of Matter Part I'!).

The answer to the first of these questions is that, with hindsight, one can see that the unification of nuclear and electromagnetic forces is much easier than the unification of gravitation and electromagnetism, so the earlier physicists had actually tackled the more difficult project. The reason for the relative ease of the transition from electromagnetism to the nuclear interactions is that both of them are theories of spin-one particles, photons and vector-mesons respectively. (This had not been appreciated at first because the vector character of the nuclear interactions had been masked at the experimental level by other phenomena, the so-called spontaneous symmetry breaking and confinement phenomena, and indeed the main advance has been to disentangle these complications). Gravitation, in contrast, is a spin-two theory in Einstein's version, and a spin-zero theory in Newton's, but in no version is it a spin-one theory (indeed if it were, gravitation would be a repulsive, rather than an attractive force) and it is the difference in spins that constituted the great obstacle in the earlier attempts at unification.

The role of the spin answers, at least to some extent, the question as to why the modern physicists have had some success in unifying the nuclear forces with electromagnetism, but it does not answer the question as to why they are so sanguine about the possibility of including gravitation. Indeed, if anything, it makes that question even more pertinent. The basis for the renewed hopes about gravitation is that in 1974, just when the neutral weak currents were being detected experimentally, a new, very remarkable, theory, called the theory of supersymmetry(2), made its appearance. The exciting feature of supersymmetry is that instead of dealing with single particles, or multiplets of particles with different <u>internal</u> quantum numbers such as nucleons of different charge (protons and neutrons) but the same spin, it deals with multiplets of particles of different spin. In fact it deals with finite multiplets of particles of different spin and since particles of spin one and two (together with spin zero, one-half and three-halves for the matter) is just what is needed for unifying electromagnetism and gravitation, one sees that supersymmetry opens up tremendous possibilities for unification. In view of this one should, perhaps, interject a word about the nature of supersymmetry.

Like all good ideas, the basic idea of supersymmetry is very simple, and, with hindsight, one is left only to wonder why it was not discovered earlier. To explain it let us recall the idea used by Dirac in 1928 in setting up his wave-equation. The idea was to take the square-root of the Laplacian operator by constructing four matrices γ_μ such that

$$\sum_\mu (\partial_\mu)^2 = \left(\sum_\mu \gamma_\mu \partial_\mu\right)^2 . \tag{1}$$

Similarly, the idea of supersymmetry is to take the square-root of the Dirac operator in turn, by constructing four operators Q^α such that

$$C^{\alpha\epsilon} \sum_\mu \gamma_\mu^{\epsilon\beta} \partial_\mu = \{Q^\alpha , Q^\beta\} \tag{2}$$

where α, β, ϵ are matrix indices and C is a constant matrix.* Under rotations, and, more generally, under Lorentz transformations, the Q^α must transform as spinors (so that their anti-commutator in (2) produces a vector ∂_μ) and this means that the Q^α must have half-odd-integer spin. Hence the Q^α can only connect states whose spin differs by one-half, and thus their very existence implies the existence of multiplets of particles whose spins are $j, j \pm 1/2, j \pm 1, \cdots$. Furthermore the fact that the right hand-side of (2) is an anti-commutator rather than a commutator means that the multiplets are finite, $j, j \pm 1/2, \cdots j \pm j_0$. But this is just what one wants for unification, since, as mentioned earlier, one needs spins zero, one-half and possibly three-halves for the matter fields, and spins one and two for the electronuclear and gravitational interactions, respectively.

Of course, to discover the existence of supersymmetry, and to construct a supersymmetric model that agrees with all the phenomonology, are two very different things, and so far a fully satisfactory model has not been found. But recent

* Incidentally, the process of taking square-roots ends at the level of supersymmetry because a further square-root would require operators of spin one-quarter and we know that no such operators exist.

supersymmetric models[3], based on so-called string theories, look quite promising, and this is why physicists are so hopeful now about gravitation. It is. perhaps. worth mentioning that in the string theories the natural scale of energy that enters is the Planck scale ($\sim 10^{18}$ nucleon masses) and since energies of this scale are far outside the range of the laboratories but do occur in cosmology, such theories would appear to turn the wheel full cycle and bring us back to astrophysics for more experimental information. For terrestial experiments we may have to wait another three hundred years!

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2) Bagger, J. and Wess, J. Supersymmetry and Supergravity (Princeton Univ. Press, 1983)
3) M. Green, J. Schwarz and E. Witten, Superstring Theory (Cambridge Univ. Press, 1987

DIAS-STP-87-48

One-dimensional spin-glasses, uniqueness and cluster properties

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Abstract: We discuss some recent results on absence of phase transitions in one-dimensional spin-glass models with polynomially decaying interactions. We comment on the probabilistic aspects and on the notion of "weak uniqueness".

§1 Introduction

Spin-glasses are among the more fashionable models of statistical mechanics. The original problem (and name) comes from the attempt to describe magnetic atoms (like Fe) which are diluted in a not too high concentration (like 5%) in a non-magnetic environment (like Au) and which interact via the long range oscillating RKKY - interaction.

The Hamiltonian this problem gives rise to is

$$H = -\sum_{i,j} \epsilon_i \epsilon_j \frac{\cos k_F (i-j)}{|i-j|^3} s_i s_j \tag{1}$$

where the quenched disorder vanishes $\epsilon_i = 0, 1$ describe the dilution and the s_i are spin variables.

Owing to the oscillating character of the cosine and the long-range character of the $\frac{1}{|i-j|^3}$ interaction, a particular spin can be subject to many competing

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forces from the other spins. The combination of randomness and "frustration" is generally modelled by Edwards-Anderson (EA) models of the form ([36])

$$H = -\sum_{i,j} \tilde{J}(i,j) s_i s_j \tag{2}$$

where the <u>site</u>-random Hamiltonian (1) is replaced by a <u>bond</u>-random Hamiltonian in which the $\tilde{J}(i\,,\,j)$ are independent random variables with a distribution which only depends on the distance $|i-j|$. Both short- and long-range EA models have been studied. They have been applied to many other areas, in particular to the theory of neural networks and to optimization problems.

Spin glasses have attracted extensive interest among physicists (for some recent reviews, see [1-4]). Up till now, it has been very difficult to obtain mathematically rigorous results on the presumed low-temperature spin-glass phase (for some recent heuristic theories, see [5,6,7]). On the other hand, during the last years there have been a number of results about the region where there is no phase transition (high temperature or low dimension), despite the possible occurrence of Griffiths singularities [8], which prevents the thermodynamic quantities to be analytic. In my contribution I will describe some of these results for long-range models, in particular in one dimension, and discuss some conceptual problems, like "weak" versus "strong" uniqueness of the Gibbs state.

§2 Results for long-range models

The models we consider have Hamiltonians

$$H = -\sum_{i,j \in Z} |i-j|^{-\alpha} J(i,j) s_i s_j \tag{3}$$

where the $J(i\,,\,j)$ are independent, identically distributed random variables. We use the symbol E for taking the average over the disorder variables $\{J(i\,,\,j)\}$. The $\{J\}$ distribution satisfies:

$$E\ J(i,j) = 0 \tag{4a}$$

and (for small t)

$$E \exp t J(i,j) = \exp 0(t^2). \tag{4b}$$

If we have a boundary condition σ outside a volume Λ we write

$$H_{\Lambda,\sigma} = \sum_{i,j\in\Lambda} \tilde{J}(i,j) s_i s_j + \sum_{\substack{i \in \Lambda \\ j \notin \Lambda}} \tilde{J}(i,j) s_i \sigma_j \tag{5}$$

$$(\tilde{J}(i,j) \equiv |i-j|^{-\alpha} J(i,j)).$$

The free energy of a volume Λ, at inverse temperature β and boundary condition σ is

$$\beta F_{\Lambda,\sigma}(\{J\},\beta) = \frac{-1}{|\Lambda|} \ln tr_\Lambda \exp \beta H_{\Lambda,\sigma}(\{J\}\{s\}_\Lambda). \tag{6}$$

The following results are known:

Theorem 1 [9, 10, 11]. If $\alpha > \frac{1}{2}$ and $\Lambda \to \infty$ in the sense of Fisher,

$$\lim_{\Lambda\to\infty} F_{\Lambda,\sigma} = \lim_{\Lambda\to\infty} E\, F_{\Lambda,\sigma} = f$$

exists, J-almost surely, and does not depend on the $\{J\}$ nor on the boundary condition σ, as long as σ is chosen independent of the $J(i,j)$.

Remark. A weak version of this result (convergence of the mean free energy) was proven in [12].

A stronger version, which weakens condition (4b), was recently proven by Zegarlinsky [13]. (He requests existence of moments up to 4th order of the $J(i,j)$. In fact, using his stability bound ([13] formula A6) and the subadditive ergodic theorem as in [10], the argument works even if only the second moment exists.).

This theorem is actually valid in any dimension (if $\alpha d > \frac{1}{2}$). The next theorem, however, is an essentially one-dimensional result.

Theorem 2 If $\alpha > 1$, J-almost surely we have the following:

a) There is no phase transition "in the weak sense". In the thermodynamic limit the Gibbs state is pure (extremal Gibbs) and does not depend on the (non-random) boundary condition σ.

b) The correlation functions calculated with respect to this Gibbs state decay with the same decay rate as the interaction.

c) The free energy is a C^{∞} function of temperature and magnetic field.

Remark 1 A weaker form of theorem 2a) (absence of symmetry breaking) was essentially proven in [11] and shortly after in a different way in [14]. The full proof of uniqueness and the observation that the arguments give a "weak sense" proof were given in [15]. The fact that weak uniqueness suffices for physics was discussed before in [16] (boundary conditions represent the experimental set-up, which does not depend on the sample). Weaker upper bounds on asymptotic correlation decay than given in 2b) were given in [15] and (for vector spins) in [17]. In its present form the theorem appeared shortly after the Heriot-Watt conference in [18].

Remark For the case $\alpha > \frac{3}{2}$, strong uniqueness (there is only one Gibbs state, whatever boundary conditions one prescribes) was proven in [19].

The C^{∞} - property and the asymptotic correlation decay were proven in [20]. For vector spins the (strong) absence of symmetry breaking and an upper bound on the asymptotic correlation decay were proven in [21] and [22].

In the case $\alpha > \frac{1}{2}$ (in general dimension d, $\alpha d > \frac{1}{2}$) high temperature results have been obtained by Fröhlich and Zegarlinsky [23, 24, 13]:

Theorem 3 Let $\alpha > \frac{1}{2}$. Then there is $\beta_0 > 0$, such that for $0 \leq \beta < \beta_0$ J-almost surely:

a) The Gibbs state is weakly unique

b) The correlation functions decay asymptotically at the same rate as the potential

c) The free energy is C^{∞}.

Remark Recently Fröhlich and Zegarlinsky have applied their methods to obtain a rigorous treatment of the high-temperature phase of the Sherrington-Kirkpatrick model [25]. This also has been done via different methods by Aizenman, Lebowitz and Ruelle [26].

§3. Some remarks about proofs; reduction to a non-random problem and weak versus strong uniqueness.

Most of the results in the former section have in common that they can be proven by reducing the spin-glass problem which has an interaction decaying as $|i-j|^{-\alpha}$ to a non-random problem with an effective interaction which decays as $|i-j|^{-2\alpha}$. The proof for this non-random problem can then be at different levels of complication, dependent on the problem at hand. The reduction is performed by successively splitting off terms from the Hamiltonian and afterwards applying a Taylor expansion or a probabilistic estimate to this term. We can use Fubini's theorem to interchange the average over one disorder variable $\tilde{J}(i,j)$ and the thermal average with respect to the modified Hamiltonian $H_{i,j} \equiv H_0 + \tilde{J}(i,j)s_i s_j$ where the term corresponding to this $\tilde{J}(i,j)$ has been subtracted. Because of condition (4) the final expression does not contain first order terms, but has only terms of second and higher order in $\tilde{J}(i,j)$.

For example, for the free energy we use

$$
\begin{aligned}
&E \ln tr \exp -(H_{i,j} + \tilde{J}(i,j)s_i s_j) \\
&= E \ln tr \exp H_{i,j} - \Big(E\tilde{J}(i,j)\, tr\, s_i s_j \exp H(i,j) = \Big)0 \\
&\qquad + 0\Big(|i-j|^{-2\alpha}\Big).
\end{aligned}
\tag{7a}
$$

(For a proof, see for example [17, appendix]).

For the thermal expectations we use, if $\tilde{J}(i,j)$ is small:

$$
\begin{aligned}
&E \frac{tr\, f \exp -(H_{i,j} + \tilde{J}(i,j)\, s_i s_j)}{tr \exp -(H_{i,j} + \tilde{J}(i,j)\, s_i s_j)} \\
&= E \frac{tr\, f \exp -H_{i,j}}{tr \exp -H_{i,j}} - \Big(E\, \tilde{J}(i,j)\, \{ \frac{tr\, s_i\, s_j f \exp -H_{i,j}}{tr \exp -H_{i,j}} \\
&- \frac{tr f \exp -H_{i,j}}{tr \exp -H_{i,j}} \times \frac{tr\, s_i s_j \exp -H_{i,j}}{tr \exp -H_{i,j}} \} \; = \Big)0 \; + 0\; \Big(|i-j|^{-2\alpha}\Big).
\end{aligned}
\tag{7b}
$$

(For a proof, see [16]).

For probabilistic estimates we use

$$E\ \frac{tr\ \left\{\exp-(H_{i,j}+\tilde{J}(i,j)s_is_j)\chi_{\tilde{J}(i,j)s_is_j>c}\right\}}{tr\ \exp\ -(H_{i,j}+\tilde{J}(i,j)\ s_is_j)}$$

$$\leq Cst\ \exp\ -\left(\frac{c^2}{|i-j|^{-2\alpha}}\right). \tag{7c}$$

For a proof see [15] or [27].

This type of estimate often turns out to be useful if one wants to apply the Borel-Cantelli lemma.

The nonrandom part of the proof can be either known (subadditivity in Theorem 1 [28], Araki's relative entropy method [11], [16], [29], [30], the Leuven energy-entropy inequalities [14] [31], the McBryan-Spencer estimates [17], [22] [32] [33] [34] to show the absence of symmetry breaking and upper bounds on correlation decay in one and two dimenions) or be developed for the problem at hand like the block spin arguments of [15] and [18] which are used to map the system onto an effective high T model (see also [27], [20] and [35] and the different polymer expansions of [18] and [23],[24] (see also [20] and [35]) which work in high temperature regions.

The problem of weak versus strong uniqueness comes in as follows. If one applies Fubini's theorem to the double integration with respect to the disorder variable $J(i,j)$ and the modified thermal average corresponding to $H_{i,j}$, this presupposes that $H_{i,j}$ does not contain any $J(i,j)$ - dependence. In particular, $H_{i,j}$ contains boundary conditions, and they should therefore be $J(i,j)$ - independent for the proof to work. For example, let us consider the interaction energy W between left and right halflines on Z, and consider the configuration to the right of the origin as the boundary condition. If $\alpha > 1$, for each choice of this boundary condition the expression $W_\sigma(\{J\},\{s\})$ is finite for each s and almost each $\{J\}$ (with respect to the J - distribution) and so is the partition function [11]

$$Z(W_\sigma) = tr_s\ \exp\ W_\sigma(\{s\},\{J\}. \tag{8}$$

However, if one allows J - dependent boundary conditions and takes the supremum of W_σ, over all boundary conditions σ, $\sup_\sigma W_\sigma(s) = \infty$ as soon as $\alpha < \frac{3}{2}$ [11]. The fact that $\sup_{s,\sigma} W_\sigma(s) < \infty$ for almost all J is the main ingredient in the strong uniqueness proof for the case $\alpha > \frac{3}{2}$ in [19], but as for the case $1 < \alpha < \frac{3}{2}$ one uses the estimates (7a,b,c), in which we have used Fubini's theorem to suppress the "bad" (large energy) configurations, one only obtains weak uniqueness.

A criterion for the absence of phase transitions is the disappearing of the Edwards-Anderson order parameter which is (formally) defined as [36]

$$Q_{EA} = E < s_i >_H^2 . \tag{9}$$

By an ergodic theorem one can replace the average over the $\{J\}$ by a spatial average over the lattice. Weak uniqueness then implies that (J - almost surely)

$$Q_{EA}^{(\text{weak})} = \lim_{\Lambda\to\infty} \sum_{i\in\Lambda} \frac{< s_i >^2_{\Lambda,\sigma}}{|\Lambda|} = 0 \tag{10a}$$

for each fixed boundary condition σ (or

$$\sup_\sigma \lim_{\Lambda\to\infty} E < s_i >^2_{\Lambda,\sigma} = 0).$$

Strong uniqueness means that [37] (J - almost surely)

$$Q_{EA}^{(\text{strong})} = \lim_{\Lambda\to\infty} \sup_\sigma \sum_{i\in\Lambda} \frac{< s_i >^2_{\Lambda,\sigma}}{|\Lambda|} = 0. \tag{10b}$$

Expression (10b) is equivalent to a thermodynamic definition which uses a replicated system.

At present there are no examples known of spin-glass models on regular lattices which are weakly but not strongly unique. However, such behaviour does occur for certain temperatures in the Bethe lattice spin-glass model [38]. Of course the Bethe lattice is somewhat pathological, as the size of the boundary is macroscopic, and also the free energy depends on the boundary condition in the thermodynamic limit, but it shows at least in principle that the two notions are really different. A technically related problem occurs in unbounded spin systems where weak uniqueness corresponds to uniqueness of "tempered" Gibbs states (see for example [39] [40]).

Summarising, we have reviewed some recent results on the absence of phase transitions for long-range spin-glass models, in particular in one dimension (a more heuristic treatment of this class of models can be found in [41]). We have discussed some common properties of their proofs and described the difference between "weak" and "strong" uniqueness.

Acknowledgements: I thank M. Campanino, J. Fröhlich, J.L. van Hemmen and E. Olivieri for different collaborations on these problems.

I gratefully acknowledge the hospitality of Professor J.T. Lewis and the Dublin Institute for Advanced Studies, where this contribution was written.

I thank Professor O. Penrose for inviting me to a stimulating conference at Heriot-Watt University.

I thank the Lady Davis Foundation for financial support.

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