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MANY FERMION GREEN FUNCTIONS AND DYNAMICAL ALGEBRA

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ABSTRACT

The mean-field Hamiltonian for a many-fermion system is an element in a Dynamical Algebra - a classical Lie Algebra. The thermal Matsubara and T=0 Green Functions are sums of products of factors determined by certain structure constants of the algebra. These describe the automorphisms of the Algebra.

In the mean-field approximation the Hamiltonian of an interacting many-fermion system is a sum of bilinear products of fermion creation/destruction operators with complex coefficients. This applies for systems such as a singlet BCS superconductor; and a coexisting charge-density-wave superconductor; and coexisting charge - and spin-density-wave superconductor. The Hamiltonian can be taken as

$$H = \sum_k H_k$$

where (k) is a wave-vector index. We define a basis (A_j) by relabelling the standard Fermi operators as $(A_1, A_2, \ldots) = (a_{k\uparrow}, a^{\dagger}_{-k\downarrow}, \ldots)$. Then $H_k = \sum_{ij} \lambda_{ij} X_{ij}$ with $X_{ij} = A_i^{\dagger} A_j$. The matrix λ with $(\lambda)_{ij} = \lambda_{ij}$ represents the complex coupling coefficients. The set of all pairs X_{ij} closes under Lie Bracket: $[X_{ij}, X_{\ell n}] = X_{im}\,\delta_{\ell j} - X_{\ell j}\,\delta_{im}$. If (i,j) = 1...N the algebra is an SO(4N), (or D series) algebra. The set of all singles (A_i) and $(A_i^{\dagger})$ plus all pairs (X_{ij}) generates an SO(4N+1) (or

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B series) algebra under Lie Bracket. Evidently the Hamiltonian H_k in the mean-field approximation is an element in the SO(4N) algebra. If we pass to a Cartan-Weyl realization of the basis, then H_k can be re-written as

$$H_k = \sum_j \mu_j h_j + \sum_{\alpha\pm} \mu_\alpha e_\alpha \qquad (1)$$

where the (h_j, e_α) are the generators in C-W form and (μ_j, μ_α) are some combinations of λ_{ij}.

The T=0 Green Function can be written, (suppressing spin) as

$$G(k,t) = -i < \Phi' | T_t \; a_k(t) \; a_k^\dagger \; |\Phi'> \qquad (2)$$

here $|\Phi'>$ is the (mean-field) ground state of H_k, and

$$a_k(t) = (\exp i H_k t) \; a_k \; (\exp - i H_k t) \qquad (3)$$

Using the (A_i) notation we find

$$a_k(t) = A_r(t) = \sum_j (e^{-it\lambda})_{rj} \; A_j \qquad (4)$$

so,

$$a_k(t) \; a_k^\dagger \equiv A_r(t) \; A_r^\dagger = \sum_j (e^{-it\lambda})_{rj} \; X_{rj}^\dagger \qquad (5)$$

Examining the above we can identify the matrix elements $(e^{-it\lambda})_{rj}$ as "structure constants" of an "Heisenberg" automorphism ϕ: $A_r \overset{\phi}{\to} \{A_r\}$ of single fermion operators onto themselves in the B algebra.

The H_k can be rotated to "diagonal" form: $U H_k U^{-1} = \sum_j \gamma_j h_j \equiv H_k'$, by means of a unitary transformation $U = \exp i (\sum_{\alpha\pm} \theta_\alpha e_\alpha)$, with coefficients θ_α. We can identify the coefficients γ_j as a <u>second</u> set of "structure constants" of an automorphism ϕ' of pairs of fermion operators onto themselves: $(h_j, e_\alpha) \overset{\phi'}{\to} (h_j)$ produced by the mapping U.

After rotation H_k is a sum of generators (h_j) of the Cartan sub-algebra of D. It is natural to label the kets $|\{\lambda\}>$ of H_k' by eigen-values of the h_j: $h_j \; |\lambda'\}> = \lambda_j' \; |\{\lambda'\}>$ where (λ') are the lowest eigen-values. In terms of the eigenfunction $|\{\lambda'\}>$ the state $|\Phi'>$ is $|\Phi'> = U^{-1} \; |\{\lambda'\}>$.

The Green Function G(k,t) can be written for t>0 as

$$-i \sum_j (e^{-it\lambda})_{rj} \quad < \{\lambda'\}| \, U X^{\dagger}_{rj} \, U^{-1} \, |\{\lambda'\}> \qquad (6)$$

But $U X^{\dagger}_{rj} U^{-1} \Rightarrow U(h_{\ell}, e_{\beta}) U^{-1}$ which is again an automorphism in D $\{h_{\ell}, e_{\beta}\} \;\Phi'\; \{h_{\ell}, e_{\beta}\}$ with structure constants (γ)

$$U(h_{\ell}, e_{\beta}) U^{-1} = \sum_j \gamma_j h_j + \sum_{\alpha\pm} \gamma_{\alpha} e_{\alpha} \qquad (7)$$

Taking the diagonal matrix element of (7) only the terms $\sum \gamma_j h_j$ contribute. Then for $t>0$

$$G(k,t) = (-i) \sum_j (e^{-it\lambda})_{rj} \sum_{\ell} \gamma_{\ell} \lambda'_{\ell} \qquad (8)$$

with a similar expression for $t<0$. This result gives the factorization of $G(k,t)$ into a sum of products of factors determined by the Dynamical Algebras (D and B), namely:

(1) Structure constants $(e^{-it\lambda})$ of ϕ;

(2) Structure constants γ_{ℓ} of ϕ';

(3) Eigenvalues $\{\lambda'\}$ of the operators h_{ℓ} of the Cartan sub-algebra.

A similar factorization applies for the T=0 Gor'kov (anomalous) function $F(k,t) = < \Phi' | \, T_t \, a_k(t) \, a_{-k} \, |\Phi'>$

The Thermal Matsubara Green Function ($T \neq 0$) is defined as

$$G(k,\tau) \equiv - < T_{\tau} \, a_k(\tau) \, a^{\dagger}_k > \qquad (9)$$

with ($\tau = it$) and in this case the thermal average is defined as

$$< \hat{O} > \equiv Tr \, (e^{-\beta H} \hat{O}) / Tr \;\; e^{-\beta H} \qquad (10)$$

A similar analysis as in the T=0 case gives ($\tau>0$).

$$G(k,\tau) = \sum_j (e^{-\tau\lambda})_{rj} \, (\sum_{\{\lambda_p\}} (\sum_p e^{-\beta E_p \lambda_p} \gamma_p \lambda_p)) / \sum_{p\{\lambda_p\}} e^{-\beta E_p \lambda_p} \qquad (11)$$

with a similar expression for $\tau<0$. Again, $G(k,\tau)$ is constructed as a sum of terms each of which is composed of factors which are symmetry determined from structure constants of automorphisms of the Dynamical Algebra and eigenvalues of the generators (h_ℓ). *Mutatis mutandis* the thermal Gor'kov anomalous function has a similar structure.

Details of the analysis and illustrations for simple BCS-like theory of superconductivity are given elsewhere[1), 2)].

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