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Symmetry Breaking Patterns in SO(n) Symmetric Theories

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1. Introduction.

In two recent papers[1,2] we have given the symmetry breaking patterns for SU(n) symmetric field theories with scalar fields in the adjoint or adjoint and fundamental representations. Theories with SO(n) symmetry are also of considerable interest, particularly because of their topologically non-trivial solutions[3], and so we would also like to have the exact form of the symmetry breaking for them. In this paper we find the breaking patterns for models with scalar fields in the adjoint, fundamental or adjoint and fundamental representatio

We will consider a general potential of the form

$$V = -\frac{\mu^2}{2}\operatorname{tr}\phi^2 - \frac{\nu^2}{2}(H,H) + \frac{a}{4}(\operatorname{tr}\phi^2)^2 + \frac{b}{2}\operatorname{tr}\phi^4 + \frac{\lambda}{4}(H,H)^2 + \alpha(H,H)\operatorname{tr}\phi^2 + \beta(H,\phi^2 H) \qquad (1.1)$$

where ϕ and H are in the adjoint and fundamental representations of SO(n) respectively, and (H,H) is the inner product for the fundamental representation (see section 2). With the definition

$$\sigma = \frac{n}{2}\left(\frac{n-1}{2}\right) \text{ for } n \text{ even (odd)} \qquad (1.2)$$

our results may be summarized as follows:

Pure Fundamental - $SO(n) \to SO(n-1)$

Pure Adjoint - $SO(n) \to SU(\sigma) \times U(1)$ for $b>0$

$\to SO(n-2) \times U(1)$ for $b<0$

Adjoint and Fundamental - This depends on whether n is odd or even.

n odd : $SO(n) \to SU(\sigma) \times U(1)$ for $b>0$, $\beta>0$

$SU(\sigma-1) \times U(1)$ for $b>0$, $\beta<0$

$SO(n-4) \times U(1)$ for $b<0$

n even : $SO(n) \to SU(\sigma-1) \times U(1)$ for $b>0$

$\to SO(n-4) \times U(1)$ for $b<0$

We also find a non-trivial renormalization group behaviour for the last case, since radiative corrections can change the sign of b. As in the case of SU(n), we believe that this reflects a real physical effect, namely that the shape of the effective potential seen by a particle depends on the energy of the particle, but does not affect the mass spectrum of the theory since this is defined specifically at zero meomentum.

The paper is organized as follows: In section two we establish the notation and take care of some mathematical preliminaries. In sections three and four we find the symmetry breaking for the pure fundamental and pure adjoint cases respectively. The case with both adjoint and fundamental scalars is considered in section five, and the renormalization group equations for all cases are given in section six.

2. Preliminaries.

In this section we give a brief description of the structure of SO(n) and some identities satisfied by its generators and we also give an explicit, non-standard basis for the generators in terms of nxn matrices. The first part of this section is based on the work of Cvitanovic[4].

The group SO(n) has generators T^i, $i=1\ldots.\ n(n-1)/2$, which may be written as nxn matrices T^i_{ab} satisfying

$$\mathrm{tr}\, T^i = 0 \quad , \quad \mathrm{tr}\, T^i T^j = \delta_{ij} \quad , \quad [T^i, T^j] = C_{ijk} T^k \tag{2.1}$$

$$T^i_{ab} T^j_{cd} = \tfrac{1}{2}\left(\delta_{ad}\delta_{cb} - \delta_{ac}\delta_{bd}\right) \qquad a,b,c,d = 1\ldots n \tag{2.2}$$

$$C_{ijk} = 2\, \mathrm{tr}\, T^i T^j T^k \tag{2.3}$$

where repeated indices are summed over. C_{ijk} is completely antisymmetric in i,j,k and there is no symmetric three-index object. One can define a completely symmetric four-index object

$$\begin{aligned} D_{ijk\ell} &= \mathrm{tr}\left\{T^i T^j T^k T^\ell\right\}_{sym} \\ &= d_{ijk\ell} + \frac{(2n-1)}{(n^2-n+4)} S_{ijk\ell} \end{aligned} \tag{2.4}$$

where $d_{iijk} = 0$ and $S_{ijk\ell} = (\delta_{ij}\delta_{k\ell} + \delta_{ik}\delta_{j\ell} + \delta_{i\ell}\delta_{jk})/3$ (2.5)

A useful identity which will be needed in calculating the radiative corrections to the theory is the formula for the symmetric contraction of two d's:

$$\left\{d_{ijmn}\, d_{mnk\ell}\right\} = \frac{(n^2-4)(n+1)(n-3)}{6(n^2-n+4)^2} S_{ijk\ell} + \frac{(2n-1)(n-5)(n+4)}{18(n^2-n+4)} d_{ijk\ell} \tag{2.6}$$

In what follows we shall need an explicit basis for the generators. There are several possibilities, the most common choice being the set J_{ij} of all pure imaginary antisymmetric nxn matrices. This has the advantage of familiarity but the disadvantage of having no diagonal elements. We shall instead use the "Spherical" basis[5], consisting of all real nxn matrices antisymmetric about the anti-diagonal. The definition of this basis varies slightly depending on whether n is even or odd, so we shall first consider n odd.

Let $n = 2\sigma+1$. Then the generators are G^a_b, $a,b = -\sigma\ldots\sigma$, and

$$[G^a_b, G^c_d] = \delta_{cb}\, G^a_d - \delta_{ad}\, G^c_b + \delta_{\bar b d}\, G^c_{\bar a} - \delta_{c \bar a}\, G^{\bar b}_d \qquad (2.7)$$

$$G^{\bar a}_{\bar b} = -G^b_a \quad , \quad (G^a_b)^\dagger = G^b_a \qquad \text{where} \quad \bar a = -a \qquad (2.8)$$

The diagonal elements are G^a_a, $a=1\ldots\sigma$. If we define e_{ab} as the matrix with a 1 at the a,b entry and zeros elsewhere, we have

$$G^a_b = e_{ab} - e_{\bar b \bar a} \quad , \qquad \begin{array}{cc} & \bar\sigma \xrightarrow{b} \sigma \\ \begin{array}{c} \bar\sigma \\ a \downarrow \\ \sigma \end{array} & \begin{pmatrix} & & \\ & \cdots & \\ & & \end{pmatrix} \end{array} \qquad (2.9)$$

For SO(3), for example, the generators are

$$G^1_1 = \begin{pmatrix} -1 & & \\ & 0 & \\ & & 1 \end{pmatrix} \quad , \quad G^1_0 = \begin{pmatrix} 0 & & \\ -1 & 0 & \\ & 1 & 0 \end{pmatrix} \quad , \quad G^0_1 = \begin{pmatrix} 0 & -1 & \\ & 0 & 1 \\ & & 0 \end{pmatrix} \qquad (2.10)$$

The elements of the fundamental representation are written in this basis as complex n-vectors

$$H^T = (h_{\bar\sigma}, \ldots, h_{\bar 1}, \sqrt{2}\, h_0, h_1, \ldots, h_\sigma)/\sqrt{2} \qquad \text{where } h_{\bar a} = h^*_a \qquad (2.11)$$

The invariant inner product is given by $(A,B) = A^T KB$ where K is the matrix with 1's along the anti-diagonal,

$$K = K^{-1} = \begin{pmatrix} & & & 1 \\ & & \cdot^{\cdot^{\cdot}} & \\ & 1 & & \\ 1 & & & \end{pmatrix} \tag{2.12}$$

$$\text{Then } (H,H) = h_0^2 + |h_1|^2 + \ldots + |h_\sigma|^2 \tag{2.13}$$

For even n everything goes as above with $\sigma = n/2$ and the index 0 omitted.

3. Fundamental Representation.

We shall first consider the case of a single multiplet in the fundamental representation. The results in this case are well known, but we include them for the sake of completeness and to show how they come about in our basis.

The potential in this case is

$$V = -\frac{\nu^2}{2}(H,H) + \frac{\lambda}{4}(H,H)^2 \tag{3.1}$$

The minimum is achieved when

$$(H_{min}, H_{min}) = \nu^2/\lambda \tag{3.2}$$

Suppose n is odd. Then from eq. (2.11) we see that we can choose H_{min} such that

$$h_0 = \sqrt{\nu^2/\lambda} \quad , \; h_a = 0 \quad \text{for } a \neq 0 \tag{3.3}$$

Since the SO(n-1) potential is simply 3.1 with the index 0 omitted, we see the remaining symmetry is SO(n-1).

If n is even the situation is slightly more complicated. Since there is no h_0, let us choose H_{min} such that

$$h_\sigma = i\sqrt{\nu^2/\lambda} \quad , \quad h_a = 0 \quad \text{for } a = 1 \ldots \sigma-1 \tag{3.4}$$

If we then define

$$\tilde{G}^a_o = (G^a_\sigma + G^a_{\bar{\sigma}})/\sqrt{2} \quad , \quad \tilde{G}^o_a = (G^\sigma_a + G^{\bar{\sigma}}_a)/\sqrt{2} \qquad \text{for } a = 1 \ldots \sigma\text{-}1 \tag{3.5}$$

we see that these $\tilde{G}$'s annihilate H_{min} and that together with the G^a_b for $|a|, |b| < \sigma$ they generate SO(n-1). Thus the symmetry breakdown is SO(n) $\to$ SO(n-1) for all n.

4. Adjoint Representation.

We now turn our attention to the case of a single multiplet in the adjoint representation. The potential in this case is

$$V = -\frac{\mu^2}{2} \operatorname{tr} \phi^2 + \frac{a}{4}(\operatorname{tr} \phi^2)^2 + \frac{b}{2} \operatorname{tr} \phi^4 \tag{4.1}$$

This is minimized by ϕ_0 satisfying

$$-\mu^2 \phi_0 + a \phi_0 \operatorname{tr} \phi_0^2 + 2b \phi_0^3 = 0 \tag{4.2}$$

Since this is a cubic equation, ϕ_0 can have at most 3 distinct eigenvalues. If we write it in diagonal form we see from equation (2.8) that the eigenvalues come in pairs of opposite sign, so the only possibilities for them are $\pm v$ or 0, and ϕ_0 must be of the form

$$\phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} -v & & & & & & \\ & \ddots & & & & & \\ & & -v & & & & \\ & & & 0 & & & \\ & & & & \ddots & & \\ & & & & & 0 & & \\ & & & & & & v & \\ & & & & & & & \ddots & \\ & & & & & & & & v \end{pmatrix} \tag{4.3}$$

where there are n_1 v's and $(n-2n_1)$ zeroes. Notice that if n is odd then there must be at least one zero.

Plugging this form for ϕ_0 into 4.1, we find

$$V = -\frac{n_1 \mu^2}{2} v^2 + \frac{a n_1^2}{4} v^4 + \frac{b n_1}{4} v^4 \tag{4.4}$$

Minimizing this with respect to v we find

$$v^2 = \frac{\mu^2}{n_1 a+b} \quad , \quad V = \frac{\mu^4}{4a}\left(\frac{b}{n_1 a+b} - 1\right) \tag{4.5}$$

We see from 4.4 that we must have $n_1a+b>0$ in order for the potential to be bounded below. There are then three cases to consider. First take a and b both positive. Then V is minimal when n_1a+b is as large as possible and so $n_1 = n/2$ for n even or $(n-1)/2$ for n odd.

The second possibility is to have $a<0$, $b>-na$ (n even) or $b>-(n-1)a$ (n odd). In this case V is minimal when n_1a+b is as small as possible and so once again we see $n_1 = n/2$ ($(n-1)/2$) for n even (odd).

The third case is $b<0$, $a>-b/2$. Here we want n_1a+b as small as possible and so $n_1=1$ for all n.

Thus we see that the symmetry breaking pattern depends only on the sign of b. The form of ϕ at the minimum is

$$b>0 \qquad \phi_0 = \frac{v}{\sqrt{2}}\sum_{a=1}^{\sigma} G^a_a = \frac{v}{\sqrt{2}}\begin{pmatrix} -1 & & & & \\ & \ddots & & & \\ & & -1 & & \\ & & & 1 & \\ & & & & \ddots \\ & & & & & 1\end{pmatrix} (n \text{ even}) \quad \text{or} \quad \frac{v}{\sqrt{2}}\begin{pmatrix} -1 & & & & & \\ & \ddots & & & & \\ & & -1 & & & \\ & & & 0 & & \\ & & & & 1 & \\ & & & & & \ddots \\ & & & & & & 1\end{pmatrix} (n \text{ odd}) \tag{4.6}$$

$$b<0 \qquad \phi_0 = \frac{v}{\sqrt{2}} G^\sigma_\sigma = \frac{v}{\sqrt{2}}\begin{pmatrix} -1 & & & & \\ & 0 & & & \\ & & \ddots & & \\ & & & 0 & \\ & & & & 1\end{pmatrix} \tag{4.7}$$

where v is given by 4.5.

To find the remaining symmetry we must find the generators which annihilate ϕ_0. Let us first consider $b>0$. Then from (2.7) we see

$$[\phi_0, G^a_b] = \begin{cases} 0 & a,b>0 \quad \text{or} \quad a,b<0 \\ \sqrt{2}\, v\, G^a_b & a\le 0,\ b>0 \quad \text{or} \quad a>0,\ b\le 0 \end{cases} \tag{4.8}$$

The Goldstone bosons are of the form G^a_b, $a,b>0$ while the remaining generators satisf

$$[G^a_b, G^c_d] = \delta_{cb}\, G^a_d - \delta_{ad}\, G^c_b \tag{4.9}$$

which is the definition of the algebra $U(\sigma) = U(1) \times SU(\sigma)$.

For $b<0$ the Goldstone bosons are G_σ^a and G_a^σ, $|a|<\sigma$. The remaining generators are G_σ^σ and G_b^a, $|a|$, $|b|<\sigma$. These form the algebra $SO(n-2) \times U(1)$.

5. Adjoint and Fundamental Representations.

We now turn our attention to the case where there are two scalar field multiplet one in the fundamental representation and one in the adjoint representation. The potential is given in eq. (1.1). The results differ slightly depending on whether n is even or odd, so we shall first consider the case where n is odd. Then we can put the fields at the minimum in the form

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} -V_\sigma & & & & & & \\ & -V_{\sigma-1} & & & & & \\ & & \ddots & & & & \\ & & & -V_1 & & & \\ & & & & 0 & & \\ & & & & & V_1 & & \\ & & & & & & \ddots & \\ & & & & & & & V_\sigma \end{pmatrix}, \quad H^T = (h_\sigma^*, 0, \ldots, \sqrt{2}h_0, 0, \ldots, h_\sigma)/\sqrt{2} \tag{5.1}$$

Define $$A = \mathrm{tr}\,\phi^2 = \sum_{i=1}^{\sigma} V_i^2 \quad , \quad B = (H,H) = h_0^2 + |h_\sigma|^2 \tag{5.2}$$

Then the potential can be written as

$$V = -\frac{\mu^2}{2}A - \frac{\nu^2}{2}B + \frac{a}{4}A^2 + \frac{\lambda}{4}B^2 + \alpha AB + \frac{b}{4}\sum_{i=1}^{\sigma} V_i^4 + \frac{\beta}{2}V_\sigma^2 |h_\sigma|^2 \tag{5.3}$$

Let us minimise this with respect to h_0. We find

$$\frac{\partial V}{\partial h_0} = h_0(-\nu^2 + \lambda B + 2\alpha A) = 0 \tag{5.4}$$

This has two solutions, $h_0=0$ and $\lambda B = \nu^2 - 2\alpha A$. If we take the second solution, we have

$$V = -\frac{\nu^4}{4\lambda} - \frac{1}{2}\left(\mu^2 - \frac{2\alpha\nu^2}{\lambda}\right)A + \frac{1}{4}\left(a - \frac{4\alpha^2}{\lambda}\right)A^2 + \frac{b}{4}\sum_{i=1}^{\sigma} V_i^2 + \frac{\beta}{2}V_\sigma^2 |h_\sigma|^2 \tag{5.5}$$

If β is negative, this is unbounded from below as $|h_\sigma| \to \infty$, which means we have chosen the wrong solution to (5.4). For positive β the minimum of (5.5) is attained when $|h_\sigma|=0$. Then the potential is of the same form as the pure adjoint model considered in section 4, and we see, using 4.6, 4.7 and 5.4,

$$b>0 \qquad V_1^2 = V_2^2 = \ldots = V_\sigma^2 = \frac{\lambda\mu^2 - 2\alpha\nu^2}{\lambda(b+\sigma a) - 4\sigma\alpha^2} \quad , \quad h_0^2 = \frac{(b+\sigma a)\nu^2 - 2\sigma\alpha\mu^2}{\lambda(b+\sigma a) - 4\sigma\alpha^2} \tag{5.6}$$

$$b<0 \qquad V_1^2 = \ldots = V_{\sigma-1}^2 = 0 \;, \quad V_\sigma^2 = \frac{\lambda\mu^2 - 2\alpha\nu^2}{\lambda(b+a) - 4\alpha^2} \;, \quad h_0^2 = \frac{(b+a)\nu^2 - 2\alpha\mu^2}{\lambda(b+a) - 4\alpha^2} \tag{5.7}$$

Now let us consider taking the first solution to (5.4), i.e. $h_0=0$. Then minimizing the potential with respect to $|h_\sigma|$ yields

$$\lambda|h_\sigma|^2 = \nu^2 - 2\alpha A - \beta V_\sigma^2 \tag{5.8}$$

$$\begin{aligned} V = & -\frac{\nu^4}{4\lambda} - \frac{1}{2}\left(\mu^2 - \frac{2\alpha\nu^2}{\lambda}\right)A + \frac{\beta\nu^2}{2\lambda}V_\sigma^2 + \frac{1}{4}\left(a - \frac{4\alpha^2}{\lambda}\right)A^2 \\ & - \frac{\alpha\beta}{\lambda}V_\sigma^2 A + \frac{b}{4}\sum_{i=1}^{\sigma} V_i^4 - \frac{\beta^2}{4\lambda}V_\sigma^4 \end{aligned} \tag{5.9}$$

If we now minimize (5.9) with respect to v_σ, we obtain

$$V_\sigma^2\left[\lambda(a+b) - (2\alpha+\beta)^2\right] = \lambda\mu^2 - (2\alpha+\beta)\nu^2 - \left(\lambda a - 2\alpha(2\alpha+\beta)\right)\sum_{i=1}^{\sigma-1} V_i^2 \tag{5.10}$$

Plugging this into 5.9 again yields a potential of the form (4.4), and so we see

$$b>0 \qquad V_1^2 = V_2^2 = \ldots = V_{\sigma-1}^2 = \left\{[\lambda b - (2\alpha+\beta)\beta]\mu^2 - [2\alpha b - a\beta]\nu^2\right\}/D(\sigma) \tag{5.11}$$

$$V_\sigma^2 = \left\{[2(\sigma-1)\alpha\beta + b\lambda]\mu^2 - [(\sigma-1)a\beta + (2\alpha+\beta)b]\nu^2\right\}/D(\sigma) \tag{5.12}$$

$$|h_\sigma|^2 = \left\{b(\sigma a+b)\nu^2 - b(2\sigma\alpha+\beta)\mu^2\right\}/D(\sigma) \tag{5.13}$$

where

$$D(\sigma) = \lambda b(\sigma a+b) - 4\alpha b(\sigma\alpha+\beta) - (\sigma a+b)\beta^2 + a\beta^2 \tag{5.14}$$

For b<0 we find $v_1^2 = \ldots = v_{\sigma-2}^2 = 0$, and $v_{\sigma-1}^2$, v^2 and $|h_\sigma|^2$ are given by equations 5.11 - 5.14 with σ replaced by 2.

There are thus two cases, depending on whether h_0 or h_σ is non-zero. (There is actually a third case, corresponding to the $v_\sigma=0$ solution to the minimization of 5.9, but one can show that this is only a local minimum and so we have not included it here). Explicit calculation of the potential shows that the absolute minimum is given by the $h_\sigma=0$ solution for b and β both positive and by the $h_0=0$ solution in all other cases.

Thus we have the following field configurations at the minimum:

$$b>0\,,\ \beta>0 \qquad \phi = \frac{v_\sigma}{\sqrt{2}}\begin{pmatrix} -1 & & & & & & \\ & \ddots & & & & & \\ & & -1 & & & & \\ & & & 0 & & & \\ & & & & 1 & & \\ & & & & & \ddots & \\ & & & & & & 1 \end{pmatrix},\ H^T = (0,\ldots,0,h_0,0,\ldots,0) \tag{5.15}$$

$$b>0\,,\ \beta<0 \qquad \phi = \frac{1}{\sqrt{2}}\begin{pmatrix} -v_\sigma & & & & & & & & \\ & -v_1 & & & & & & & \\ & & \ddots & & & & & & \\ & & & -v_1 & & & & & \\ & & & & 0 & & & & \\ & & & & & v_1 & & & \\ & & & & & & \ddots & & \\ & & & & & & & v_1 & \\ & & & & & & & & v_\sigma \end{pmatrix},\ H^T = (h_\sigma^*,0,\ldots,0,h_\sigma)/\sqrt{2} \tag{5.16}$$

$$b<0 \qquad \phi = \frac{1}{\sqrt{2}}\begin{pmatrix} -v_\sigma & & & & & & \\ & -v_{\sigma-1} & & & & & \\ & & 0 & & & & \\ & & & \ddots & & & \\ & & & & 0 & & \\ & & & & & v_{\sigma-1} & \\ & & & & & & v_\sigma \end{pmatrix},\ H^T = (h_\sigma^*,0,\ldots,0,h_\sigma)/\sqrt{2} \tag{5.17}$$

The symmetry groups of the minumum field configurations can be seen by inspection (or by using the procedure given at the end of section four), and we find the results given in the introduction.

If n is even everything goes as above with the index 0 omitted. This means that there is no analogue of eq. 5.15, and so the minimum for b>0 is of the form of eq. 5.16 (with the zero eigenvalue of ϕ omitted) for all β.

6. One-loop Renormalization Group Equations.

The one-loop renormalization group equations for our model can be easily calculated using the effective potential technique of Fujimoto, O'Raifeartaigh and Parravicini[6]. Defining

$$\phi_i = \mathrm{tr}\, \phi T^i \tag{6.1}$$

we have

$$V = -\frac{\mu^2}{2}\phi^2 - \frac{\nu^2}{2}(H,H) + \frac{\hat{a}}{4}(\phi^2)^2 + \frac{b}{2} d_{ijk\ell}\, \phi_i \phi_j \phi_k \phi_\ell + \frac{\lambda}{4}(H,H)^2 \tag{6.2}$$

$$+ \alpha \phi^2 (H,H) + \beta \phi_i \phi_j (H, T^i T^j H)$$

where $\hat{a} = a - \frac{2(2n-1)}{n^2 - n + 4} b$ and d_{ijkl} is defined in eq. 2.4.

We can think of ϕ_i and H_a as forming one $n(n+1)/2$ component object θ_i. Then the one-loop renormalization group equation for V is

$$\mu \frac{dV}{d\mu} = \frac{1}{64\pi^2}\, \mathrm{tr}(V'')^2 \qquad \text{where } V''_{ij} = \frac{\partial^2 V}{\partial\theta_i \partial\theta_j} \tag{6.3}$$

The equations for the various coupling constants are obtained by matching coefficients of ϕ and H. Using the identities given in section two, we find

$$\frac{d\hat{a}}{dt} = (N+8)\hat{a}^2 + \frac{6(n^2-4)(n+1)(n-3)}{(n^2-n+4)^2} b^2 + 4n\alpha^2 + 8\alpha\beta + \frac{4(2n-1)}{n^2-n+4}\beta^2 \tag{6.4}$$

$$\frac{db}{dt} = 12\hat{a}b + \frac{(2n-1)(n-5)(n+4)}{n^2-n+4} b^2 + 2\beta^2 \tag{6.5}$$

$$\frac{d\lambda}{dt} = 4N\alpha^2 + 4(n-1)\alpha\beta + (n-1)\beta^2 + (n+8)\lambda^2 \tag{6.6}$$

$$\frac{d\alpha}{dt} = (N+2)\hat{a}\alpha + \frac{1}{2}(n-1)\hat{a}\beta + (n+2)\lambda\alpha + \lambda\beta + 8\alpha^2 + 4\beta^2 \tag{6.7}$$

$$\frac{d\beta}{dt} = 2\beta(\hat{a} + \lambda + 8\alpha + 2n\beta) \tag{6.8}$$

$$\frac{d\mu^2}{dt} = (N+2)\hat{a}\mu^2 + 2(n\alpha+\beta)\nu^2 \tag{6.9}$$

$$\frac{d\nu^2}{dt} = (2N\alpha + (n-1)\beta)\mu^2 + (n+2)\lambda\nu^2 \tag{6.10}$$

where $t = (1/16\pi^2)\,\ell n\mu$ and $N = n(n-1)/2$.

The most interesting feature of these equations is that they can change the sign of b and thus the effective symmetry of the potential. From (6.5) and (6.8) we see that if $\beta=0$ at some $t=t_0$, then the renormalization group will not change it as t varies and since the change in b is proportional to b itself, b can never change sign. For non-zero β, however, the β^2 term in 6.5 can cause b to change sign. We interpret this as meaning that the effective shape of the potential for interacting particles is momentum dependent. The symmetry properties of the vacuum are unambiguous, however, because they are defined at a particular value (zero) of the momentum.

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