DIAS-STP-80-25

Comments on SU(9) Grand Unified Theory.

Y. Fujimoto,*
School of Theoretical Physics,
Dublin Institute for Advanced Studies,
10 Burlington Road, Dublin 4.

and

P. Sodano,
Istituto di Fisica,
Universita di Salerno,
Salerno,
Italy.

* Present address: ICTP, Trieste, Italy.

Abstract

Frampton's SU(9) model is detailed considering it as a G.U.Th. with SU(4) horizontal symmetry. We find a correlation among neutrino, horizontal gauge boson and new fermions' masses. With neutrino mass around 10ev horizontal gauge boson is estimated to be as heavy as 6×10^{10}Gev. The theory also contains new charged current processes, B + L conserving and $\Delta L = 2$.

1. Introduction

The grand unified theories(G.U.Th.) have been proposed to unify strong, weak and electromagnetic interactions. Some of the models[1] not only retain the phenomonologically successful features of SU(3) x SU(2) x U(1) gauge theory but also predict very interesting phenomena such as proton decay and baryon number asymmetry in the universe. The present form of G.U.Th. would, however, be incomplete in the sense that gravitational interaction is not unified and that the fact of light family replication cannot be explained. The aim of the present paper is to introduce a local horizontal symmetry among families and unify it with a conventional G.U.Th. and study its consequences. Although there is as yet no clear indication of the need of a local horizontal symmetry we propose it as one way of understanding the family structure. Then the first question to be asked is what is the right horizontal symmetry.[2] It was once assumed to be $SU^H(3)$[3] (H denotes "horizontal") the motivation for which was to incorporate only three light families into the fundamental representation. On the other hand if we believe in the sequence of successful physical gauge symmetry,

$$U(1) \subset SU^W(2) \subset SU^c(3) \quad \ldots\ldots\ldots ,$$

$SU^H(4)$ may be the next symmetry to be exploited. By adding $SU^H(4)$ on top of SU(5) we arrive at SU(9) G.U.Th. In this paper we study a SU(9) G.U.Th. proposed by P. Frampton.[4] . The main purpose of our analysis has been to present the general features of G.U.Th. with a horizontal symmetry by taking explicit examples of G.U.Th. By comparing SU(8)[3] and SU(9) models one will see that most of the features are shared by both models but they are very different so far as the fermion mass spectrum is concerned. We pay particular attention to the fermion mass since it could serve the purpose of squeezing down the candidates of G.U.Th. In fact it is pointed out[3] that SU(8) vector-like model is unlikely to survive unless a novel way of producing mass is found. The fermion mass is, in this paper, assumed to arise in the standard manner i.e. through Yukawa couplings of fermions and scalars with spontaneous breaking. Yukawa coupling constants are assumed to be smaller than the weak gauge coupling constant in the lower energy region to secure the asymptotic freedom in the region. (As we shall see later the asymptotic freedom is lost in this model in the high energy limit.) The masses of weak gauge boson 85 Gev and of $SU^H(4)$ gauge boson, larger that 10^4Gev. serve as very effective constraints on the fermion masses. We point out that small neutrino mass appears naturally and also that the dynamical creation of mass is possible in this model. (See section III).

Other features of the model such as renormalization effects and charged currents will be studied in section IV and V respectively.

II. Model

1. Charge Operator

To avoid exotic fermions Q (= charge operator) is chosen to be,

$$Q = \mathrm{diag}\,(-\tfrac{1}{3},\ -\tfrac{1}{3},\ -\tfrac{1}{3},\ 1,\ 0,\ 0,\ 0,\ 0) \tag{1}$$

for the fundamental representation. This assignment guarantees $\sin^2\theta_W = \frac{3}{8}$ at the symmetric limit.

2. Fermions

With left and right handed parts counted separately there are one hundred and sixty five fields,

$$165 = 84_\ell + 9 \times 9r. \tag{2}$$

Reasonable assignment is as follows

9 x 9r

$$(d_\alpha,\ e^c,\ \nu_e^c,\ F_{1i}) \tag{3-1}$$
$$(s_\alpha.\ \mu^c.\ \nu_\mu^c.\ F_{2i}) \tag{3-2}$$
$$(b_\alpha.\ \tau^c.\ \nu_\tau^c.\ F_{3i}.) \tag{3-3}$$
$$(q_{1\alpha}.\ \ell_1^c.\ \nu_1^c.\ F_{4i}.) \tag{3-4}$$
$$(q_{2\alpha}.\ \ell_2^c.\ \nu_2^c.\ F_{5i}.) \tag{3-5}$$
$$(q_{3\alpha}.\ \ell_3^c.\ \nu_3^c.\ F_{6i}) \tag{3-6}$$
$$(q_{4\alpha}.\ \ell_4^c.\ \nu_4^c.\ F_{7i}) \tag{3-7}$$
$$(q_{5\alpha}.\ \ell_5^c.\ \nu_5^c.\ F_{8i}) \tag{3-8}$$
$$(q_{6\alpha}.\ \ell_6^c.\ \nu_6^c.\ F_{9i}.) \tag{3-9}$$

$$84_\ell = (4.10) + (1.\overline{10}) + (6.5) + (\bar{4}.1) \quad \text{(with respect to } SU^H(4) \times SU(5)\text{)}$$

$$(4.10) \quad \left[u^{\alpha c} \begin{pmatrix} u_\alpha \\ d_\alpha \end{pmatrix} e^c \right] \quad \left[t^{\alpha c} \begin{pmatrix} t_\alpha \\ b_\alpha \end{pmatrix} \tau^c \right] \qquad (4\text{-}1)$$

$$\left[c^{\alpha c} \begin{pmatrix} c_\alpha \\ s_\alpha \end{pmatrix} \mu^c \right] \quad \left[t'^{\alpha c} \begin{pmatrix} t'_\alpha \\ b'_\alpha \end{pmatrix} \tau'^c \right]$$

$$(1.\overline{10}) \quad \left[t'_\alpha \begin{pmatrix} t'^{\alpha c} \\ b'^{\alpha c} \end{pmatrix} \tau' \right] \qquad (4\text{-}2)$$

$$(6.5) \quad \begin{matrix} (q_{1\alpha} \;\; l_1 \;\; \nu_1) & (q_{4\alpha} \;\; l_4 \;\; \nu_4) \\ (q_{2\alpha} \;\; l_2 \;\; \nu_2) & (q_{5\alpha} \;\; l_5 \;\; \nu_5) \\ (q_{3\alpha} \;\; l_3 \;\; \nu_3) & (q_{6\alpha} \;\; l_6 \;\; \nu_6) \end{matrix} \qquad (4\text{-}3)$$

$$(\bar{4}.1) \quad F^i_{10} \qquad (4\text{-}4)$$

In the above table α (= 1,2,3) and i (= 1,2,3,4) are colour and $SU^H(4)$ indices, respectively, and C denotes charge conjugation. As for Fpi (p= 1 ~ 10) they could be all independent or some of them could be anti-particles. In the former case they are all one-handed and they are massless in the ordinary sense, i.e. Dirac mass is zero. But as will be seen in section III they can acquire a large Majorana mass and thus does not contradict the expermiental result and and cosmological consideration which predicts the number of massless fields to be not more than four. (Majorana mass was first mentioned in the context of G.U.Th. by Gell-Mann et al.[5] and has recently been discussed by many people [6,7].

3. Gauge fields

New gauge bosons appear besides those of SU(5).

New gauge bosons	$SU^C(3) \times SU(3) \times SU^H(4)$	charge	
$(A_\mu)^i_j$	(1, 1, 15)	0	
$(Z_\mu)_{i\alpha}$	(3, 1, 4)	$-\frac{1}{3}$	15)
$\begin{pmatrix} V_\mu^+ \\ V_\mu^0 \end{pmatrix}_i$	(1, 2, 4)	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	
B'_μ	(1, 1, 1)	0	

A_μ are $SU^H(4)$ gauge bosons and cause the change of flavour. Z_μ and $V_\mu^\pm$ are charged bosons and mediate B + L conserving and $\Delta L = 2$ processes. Being a singlet with respect to $SU^C(3) \times SU(2) \times SU^H(4)$ B'_μ behaves in a similar manner as B_μ of the SU(5) model. But we note there is no new mixing angle with respect to weak and electromagnetic interaction because the charge operator is chosen as in eq. (1).

4. Breaking pattern and gauge boson mass.

We assume the following breaking pattern

$$SU(9) \rightarrow SU(5) \times SU^H(4) \times U(1) \rightarrow SU^C(3) \times SU(2) \times SU^H(4) \times U(1) \times U(1) \rightarrow SU^C(3) \times SU(2) \times U(1) \rightarrow SU^C(3) \times U(1) \tag{6}$$

Step A and B could be realized by two adjoint scalar multiplets, ϕ_1 and ϕ_2,

$$\phi_1 = v_1 \,\mathrm{diag}(1, 1, 1, 1, 1, -\tfrac{5}{4}, -\tfrac{5}{4}, -\tfrac{5}{4}, -\tfrac{5}{4}) \tag{7-1}$$

$$\phi_2 = v_2 \,\mathrm{diag}(1, 1, 1, -\tfrac{3}{2}, -\tfrac{3}{2}, 0, 0, 0, 0) \tag{7-2}$$

where

$$10^{15}_{\mathrm{Gev}} \simeq v_1 < v_2 < 10^{19}_{\mathrm{Gev}} \tag{7-3}$$

Step C and D are assumed to be effected by the scalar multiplets which also couple with fermion bi-linears (adjoint scalars do not couple with fermion bi-linears).

In this setting we have

$$10^{14}_{\mathrm{Gev}} \simeq m_{X_\mu(Y_\mu)} < m_{V_\mu(Z_\mu)} < 10^{19}_{\mathrm{Gev}} \tag{8}$$

and

$$m_{A_\mu(B_\mu)} < m_{Z_\mu(V_\mu)}$$

Also the suppression of flavour changing neutral currents puts lower bound for A_μ

$$m_{A_\mu} > 10^4\,\mathrm{Gev}. \tag{9}$$

III Fermion mass

We start the discussion on the fermion mass spectrum with the constraints imposed by the mass of weak and $SU^H(4)$ gauge bosons. It should be noted that the scalar components which contribute to the fermion mass also contribute to the gauge boson mass. Namely any V.E.V. of SU(2) ($SU^H(4)$) non-singlet scalar components contribute to the weak ($SU^H(4)$) gauge boson mass and thus its value is restricted. This observation greatly simplifies the analysis and we can easily tell which fermions can be heavy or light. In a realistic model we would expect that the mass matrix is so polarized that many or all of the new fermions are much heavier than familiar ones. We restate below the criteria[3] for extracting heavy fermions in the present context.

" Fermions can be heavy if they are SU(2) singlet and $SU^H(4)$ non-singlet or if both left- and right-handed parts are SU(2) non-singlet and their fermion bi-linears are SU(2) singlet and $SU^H(4)$ non-singlets."

In practice our criteria presumably leads to the same conclusion as H. Georgi's[8] but ours apply to any type of models besides SU(N) chiral model. Its content becomes clearer once we write out fermion bi-linears and decompose them with respect to $SU^H(4) \times SU(5)$ as is presented in the Appendix. By making use of the table in the Appendix we obtain the following results. Hereafter, in this section numbers, (1), (9), etc., refer to those in the Appendix.

1. F_{pi} $(p = 1 \sim 10,\ i = 1 \sim 4)$

F_{pi} becomes heavy by acquiring a large Dirac and/or a Majorana mass through scalar components numbered as (1), (2), (4), (5), (9), (10) in the Appendix.

2. $\nu_s,\ l_s$ $(s = 4 \sim 9)$

Through (12) ν_s and l_s obtain a large Dirac mass.

3. t', b', τ'

(6). gives a large Dirac mass for t', b' and τ'.

At this stage all familiar fermions are massless. To make them massive SU(5) non-singlet scalar components should develop a small V.E.V.

4. u, c, t,

u, c, t, become massive through (7) and (8).

5. d, s, b, e, μ, τ

(11) makes them massive. If flavour mixings are neglected we recover the familiar SU(5) mass relation

$$\frac{m_e}{m_d} = \frac{m_\mu}{m_s} = \frac{m_\tau}{m_b} \quad . \tag{10}$$

6. ν_e, ν_μ, ν_τ

These neutrinos are one-handed and do not have Dirac mass. We point out however that they may acquire very small Majorana masses through (3). ((3) is SU(5) non-singlet and its V.E.V. is small). The reason is the following. F_i may acquire large Majorana mass through (1) (2) (4) (5) (9) (10) and (3) causes mixings among ν_e (ν_μ, ν_τ) and F_i. Then forgetting about other possible mixings we have for example, a mass matrix[6,7],

$$\begin{array}{c|cc} & \nu & F_i \\ \hline \nu & 0 & m \\ F_i & m & M \end{array} \tag{11}$$

m (M) comes from (3) ((1), (2), (4), (5), (9), (10),) and thus is small(can be very large). If $M \gg m$ ν_e acquires a Majorana mass $\sim \frac{m^2}{M}$.

If neutrino mass is less than 10ev, then we obtain a lower bound for $SU^H(4)$ gauge boson mass, $\gtrsim 6 \times 10^{10}$Gev. In such case all new fermions become superheavy with the mass around 10^9Gev or larger.

Next we comment on the scalars to be used. To break SU(2) only 5 dim. representation(with respect to SU(5) sub-group) is used to maintain the normal mixing relation between photon and neutral weak gauge boson. One result associated with this choice is that the neutrinos are necessarily massive in this model because, as the Appendix shows, the scalar component which is 5 dim. with respect to SU(5) is not singlet with respect to SU(4) and thus neutrinos always mix with Fs. Also it should be mentioned that 84 dim. scalar multiplet does not couple with 84 x 84 fermion bi-linear and 36 dim. scalar multiplet does not couple with 9 x 9 in case two 9s are identical. This is due to the anti-commuting nature of fermion fields and is explained in ref. [9].

We may, incidentally, turn around the way of thinking on creating fermion mass and look for the possibility of cf creating it dynamically [10]. This possibility is not realized in an arbitrary model but interestingly in this particular model it is. One could forbid some of 9_Rs to couple with 84_L, so that there is no direct coupling between left and right handed parts. Then such fermions can become massive only through radiative corrections and it is known that the radiative corrections can give sizeable contributions [10]. This way of creating mass is very attractive because it will naturally explain the smallness of ordinary fermion masses. The actual implementation of the idea is rather involved and will be discussed elsewhere.

IV Asymptotic freedom and Renormalization effects

We summarize in this section the behaviour of the effective coupling constants. Their behaviour depends on β which is given at one loop level[11] by

$$\beta = -\frac{11}{3}G + \frac{4}{3}F + \frac{1}{6}S \qquad (12)$$

where

G, F and S denote gauge, fermion and scalar contributions respectively.

In our model we have, neglecting scalars,

$$\begin{aligned} \beta(SU(2)) &= \frac{8}{3} \\ \beta(SU^c(3)) &= -1 \\ \beta(SU^H(4)) &= -\frac{14}{3} \end{aligned} \qquad (13)$$

Three comments are due.

1. Without scalars Q.C.D. is asymptotically free, so is the entire theory in the high energy limit. However, if scalars are included asymptotic freedom is lost because 1050, 2520 or 3402 must be introduced. So far as we have studied there is no asymptotically free G.U.Th. based on the single group with its rank larger than seven.
2. Crossing of coupling constants can take place. Suppose the breaking pattern is stepwise

$$SU(9) \to SU(5) \times SU^H(4) \times U(1) \to SU(3) \times SU(2) \times SU^H(4) \times U(1) \times U(1) \qquad (14)$$

and mass of $SU^H(4)$ gauge bosons is less than 10^{10} Gev then $SU^H(4)$ coupling constant crosses with both $SU^C(3)$ and $SU(2)$ coupling constants. Crossing has been noted in SU(8) model.[3] There, $SU^H(3)$ coupling constant crosses with that of SU(2) but not $SU^C(3)$. If the breaking pattern is

$$SU(9) \rightarrow SU^C(3) \times SU(6) \longrightarrow SU^C(3) \times SU(2) \times SU^H(4) \times U(1) \times U(1) \qquad (15)$$

then $SU^C(3)$ and $SU(2)$ cross each other.

3. The prediction of mixing angle and g_s (=gluon coupling constant) at the present energy remains almost the same as that of SU(5) model even though each effective coupling constant behaves significantly different from those of SU(5).

V Charged Currents

The feature of G.U.Th. which is of great physical interest is the existence of the processes which break both B (=baryon number) and L (=lepton number). SU(5) model contains B-L conserving processes mediated by $X_\mu^{\pm 4/3}$ $Y_\mu^{\pm 1/3}$ charged super-heavy vector bosons and they lead to proton decay . Also it is hoped that they may explain the baryon number asymmetry in the universe

In our model new charged bosons $V_\mu^\pm$ and $Z_\mu^{\pm\frac{1}{3}}$ appear besides $X_\mu^{\pm\frac{4}{3}}$ and $Y_\mu^{\pm\frac{1}{3}}$. Their couplings with fermions turn out to be as follows

	9r	84ℓ
V_μ^+	$\bar{l}^c F_i$ and/or $\bar{l}^c F_i^c$	$\bar{l}^c F_{1\sim 9}$, $\bar{u} d$, $\bar{t}' d$
$Z_\mu^{-\frac{1}{3}}$	$\bar{d} F_i$ and/or $\bar{d} F_i^c$	$\bar{d} F_{1\sim 9}$, $\bar{u} l^c$, $\bar{d} \nu^c$, $\bar{u}^c d$.

where F, l, ν, d and u denote $F_1 \sim F_9$, $l_1 \sim l_6$ and e, μ, τ, $\nu_1 \sim \nu_6$ and ν_e, ν_μ, ν_τ, (d, s, b) and (u, c, t), respectively. And/or is due to the arbitrariness of particle and anti-particle assignment. The above table shows the existence of new type of processes. V_μ mediate $\Delta L = 2$ process and Z_μ mediate B + L conserving and $\Delta L = 2$ processes at the lowest order. These processes have been found in SU(8) model [3] and also in SO(18) and E(8) models. We expect it to be a general feature of G.U.Th. with a horizontal symmetry. These new processes may affect the estimate of B-asymetry [12] in the universe although we have not performed numerical calculation.

V. Summary and Comments

We discussed consequences of SU(4) horizontal symmetry by taking Frampton's model as an example. The model is similar to SU(8) model so far as the charged current structure and the renormalization effects are concerned. We have noted $\Delta L = 2$ and B + L conserving processes. The difference shows up in the fermion mass spectrum. SU(9) model has three light families whereas SU(8) model contains five. The model also provides the possibility of creating small Majorana mass for neutrinos and of creating light fermion masses dynamically. Majorana mass of neutrino may resolve the problem of missing mass and in the universe and dynamical creation of small mass would solve the problem of families in a very appealing manner, even though it is achieved at the cost of introducing a local horizontal symmetry and superheavy fermions. The picture emerging from the analysis is; there are superheavy bosons and fermions and their existence is reflected on the light fermions and perhaps weak gauge bosons in the form of small masses, flavour mixings, Weinberg angle etc..

Our analysis is admittedly incomplete. In discussing symmetry breaking and fermion mass spectrum we assumed, without proof, that certain components of scalars develop suitable vacuum expectation values. This problem of gauge hierarchy becomes more difficult as we go to larger groups since the theory could undergo multi-stage breaking. We would like to come back to the problem in the future.

Acknowledgement

One of us (Y.F.) would like to thank Drs. D. Nanopoulas, J. Ellis, G.t'Hooft and S.L. Glashow for brief discussions. He also deeply thanks members at D.I.A.S. and Istituto di Fisica, University of Salerno for hospitality where the work was done.

Appendix

I. 9 x 9 = 36 + 45

	(4.1)	(1.5)
(4.1)	(6.1) [1] (10.1) [2]	(4.5)[3]
(1.5)		(1.10) (1.15)

II. 84 x 84 = $\overline{84}$ + 1050 + 2520 + 3402

	(4.10)	(6.5)	(1.$\overline{10}$)	(4.1)
(4.10)	(6.5)[7] (10.5)[8] (6.45) (10.45) (6.50) (10.50)	(4.10) (20.10) (4.40) (20.40)	(4.1)[6] (4.24) (4.75)	(1.10) (15.10)
(6.5)		(1.10) (20.10) (1.15) (20.15) (15.10) (15.15)	(6.10) (6.40)	(4.10) (20.5)
(1.10)			(1.1) (1.24) (1.75)	(4.10)
(4.1)				(6.1)[4] (10.1)[5]

III. $\overline{9}$ x 84 = 36 + $\overline{720}$

	(4.10)	(6.5)	(1.$\overline{10}$)	($\overline{4}$.1)
($\overline{4}$.1)	(1.10) (15.10)	(4.5) (10.5)	(4.10)	(6.1)[9] (10.1)[10]
(1.$\overline{5}$)	(4.5)[11] (4.45)	(6.1)[12] (6.24)	(1.10) (1.40)	(4.5)

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