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Statics and Dynamics of Classical Yang-Mills-Higgs Systems: Some Recent Developments*

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ABSTRACT

Some classical Yang-Mills-Higgs solutions, all characterized by an underlying nontrivial topology, are studied. First, the explicit construction of magnetic n-pole solutions is briefly reviewed. Second, theories which allow for noncontractible loops and static saddle points which result from this type of nontrivial topology are exhibited. We then turn to time-dependent solutions. Here, we first state the underlying ideas for the description of the scattering of slowly-moving monopoles. Finally, Segal's theorem is discussed and the existence proofs for time-dependent vortices and monopoles, which apply to the equations of motion without any approximation, are outlined.

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I. Introduction

We have good reason to believe that gauge theories can be used to describe all the known interactions. In particular, we believe, that U(1) gauge theory describes electromagnetism, the gauge theory of general relativity describes gravitation, U(2) gauge theory describes the electroweak force and SU(3) gauge theory describes the strong force. Some physicists even think that a grand unified theory based on the gauge group SU(5) or SO(10) or E(6) or another group unifies the description of the electroweak and the strong force. Although the strength of our faith varies considerably from electromagnetism to grand unified theories there is no doubt that gauge theories deserve a careful study.

In these lectures, we will discuss, in particular those gauge theories for which the Higgs mechanism has been invoked to explain the short range of the forces. This mechanism has been used in the phenomenological U(1) gauge theory of superconductivity by Landau and Ginzburg[1], in the U(2) gauge theory of electroweak interactions by Weinberg[2], and by Salam[3] and in grand unified theories[4]. We will study all of these Yang-Mills-Higgs theories.

Given the importance of gauge theories it is also natural to study their classical solutions. As far as electromagnetism is concerned, this is an integral part of the education of any physicist, and its relevance cannot be disputed. As for nonlinear gauge theories, the relevance of classical solutions is less apparent. On the other hand, however, these theories admit soliton-like solutions which cannot be present in a linear theory, such as vortices in the Landau-Ginzburg model, saddle points in the SU(2) part of the bosonic Weinberg-Salam model, or monopoles in grand unified theories.

In the following, I will address myself to all of these solutions, discussing static as well as time-dependent versions. Naturally, I will concentrate on work in which I am myself involved at the moment, that is the study of static saddle points, and the theory of global existence proofs for time-dependent solutions. Related, but by no means less relevant, work can only be sketched and used as a point of reference in the small amount of time available.

II. Yang-Mills-Higgs theory

The dynamical variables of Yang-Mills-Higgs theories are the orbits of gauge potentials and Higgs fields under the action of a gauge group G. This means the following: We are given a compact Lie group G of dimension r, gauge potentials

$$A^a_\mu(x); \quad \mu = 0,1,\ldots,D; \quad a = 1,\ldots,r,$$

and real Higgs fields

$$\phi_s(x); \quad s = 1,\ldots,\ell.$$

Under a local gauge transformation $g(x) \in G$, these fields transform as

$$\begin{aligned} A_\mu := A^a_\mu T_a &\to A'_\mu = gA_\mu g^{-1} + i\partial_\mu g g^{-1}, \\ \vec{\phi} &\to \vec{\phi}' = g\vec{\phi}, \end{aligned} \tag{2.1}$$

where T_a are the hermitean generators of G, and g denotes the group element in its fundamental or ℓ-dimensional representation, respectively. Because we assume that the physics is independent of gauge transformations, the relevant variables are the group orbits rather than the fields themselves.

The Yang-Mills-Higgs Lagrangian is

$$\mathcal{L} = -\tfrac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a + \tfrac{1}{2}(D_\mu\phi)^s(D^\mu\phi)_s - \frac{\lambda}{4}(|\phi|^2-1)^2, \tag{2.2}$$

where

$$F_{\mu\nu} = F^a_{\mu\nu}T_a = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu,A_\nu] \tag{2.3a}$$

are the gauge fields, and

$$D_\mu\vec{\phi} = \partial_\mu\vec{\phi} + iA_\mu\vec{\phi} \tag{2.3b}$$

is the covariant derivative of $\vec{\phi}$. Our metric is diag $(+,-,\ldots,-)$, and we have chosen a special isotropic Higgs potential which will be sufficient for our purpose. The Lagrangian (2.2) is invariant under the gauge transformations (2.1). The corresponding equations of motion read

$$\begin{aligned} D_\mu D^\mu\vec{\phi} &= \lambda\vec{\phi}(1-|\phi|^2), \\ D_\nu F^{\mu\nu} &= -\frac{i}{4}[(D^\mu\vec{\phi})^+ \cdot T_a\vec{\phi} - \vec{\phi}^+ \cdot T_a D^\mu\vec{\phi}]T_a. \end{aligned} \tag{2.4}$$

We want to find topologically nontrivial solutions to these equations.

Thus, at this point we must discuss the topology of Yang-Mills-Higgs theories. We will do this without going into mathematical details. (For a mathematically rigorous discussion see e.g. Jaffe and Taubes[5].) Let us look at static finite-energy configurations in the $A_0=0$ gauge. For these configurations, $D_i\phi$ $(i=1,\ldots,D)$ and $(|\phi|^2 - 1)$ are square-integrable. Let us furthermore assume that ϕ is defined and continuous at infinity. Then $|\phi|$ goes to 1 at infinity which defines the map

$$\phi_\infty : S^{D-1} \to S^{\ell-1}.$$

In special cases, this map is nontrivial, i.e., there are configurations which cannot be continuously deformed into each other.

The cases we will study in detail are (A) the Landau-Ginzburg theory in 2 space dimensions ($G=U(1)$, $\ell=2$, $D=2$) and (B) SU(2) gauge theory with a Higgs field in the adjoint representation in 3 space dimensions ($G=SU(2)$, $\ell=3$, $D=3$). The topologically nontrivial maps are

$$\text{(A)} \quad \phi_\infty : S^1 \to S^1,$$

and

$$\text{(B)} \quad \phi_\infty : S^2 \to S^2,$$

respectively. They fall into classes labelled by their winding number n.

We assume furthermore that $D_i\phi$ vanishes fast enough with the necessary smoothness conditions at infinity, so that we can integrate the equation $(D_i\phi)_\infty=0$. This yields

$$\Phi_\infty(\hat{x}) = P(\exp i \int_{\hat{x}_0}^{\hat{x}} A_i dx^i)\Phi_\infty(\hat{x}_0) = : \omega\Phi_\infty(\hat{x}_0), \tag{2.5}$$

which defines a map

$$\omega : S^{D-1} \to G/H,$$

where H is the little group of $\Phi_\infty(\hat{x}_0)$. We can therefore study the homotopy group $\pi_{D-1}(G/H)$ and its homotopy classes $[(A,\Phi)]$. If G acts transitively on $S^{\ell-1}$, then $[(A,\Phi)]$ is equal to the homotopy class $[\Phi]$ defined above.

With the above conditions on $D_i\Phi$ we can furthermore show that the winding number in case (A) is proportional to the magnetic flux:

$$\text{(A)} \quad n = \frac{1}{2\pi} \lim_{r\to\infty} \int_{|x|<r} d^2x F_{12}, \tag{2.6a}$$

and that the winding number in case (B) is proportional to the magnetic charge:

$$\text{(B)} \quad n = \frac{1}{4\pi} \lim_{r\to\infty} \int_{|x|=r} d\sigma^i \,\hat{\Phi}_a B^i_a. \tag{2.6b}$$

The corresponding configurations are called vortices and monopoles, respectively.

In the following, we will discuss monopole solutions as well as vortex solutions. We will also become acquainted with a different type of nontrivial topology. In this case, it is not the topology of configurations but the topology of families of configurations which is relevant. This nontrivial topology leads to saddle points and because it is less familiar it will be discussed in more detail later.

III. Construction of magnetic monopole solutions

Let me remind you of one line of essential steps which led to the construction of magnetic monopole solutions: We restrict our attention to case (B) now, i.e., we discuss SU(2) gauge theory with a Higgs field in the adjoint representation. We furthermore let λ go to zero while keeping the boundary condition $|\Phi| \to 1$ for $r\to\infty$. This is the Bogomol'nyi-Prasad-Sommerfield (BPS) limit[6)7)]. Our assumptions simplify the search for static solutions in the $A_0=0$ gauge to such an extent that we will be able to construct magnetic monopole solutions explicitly.

Because of our assumptions we can formulate our problem as a problem of finding time-independent solutions to pure SU(2) gauge theory on $\mathbb{R}^4$. In fact, all we have to do is to identify the gauge potential A_4 with Φ. Then, we see that the Lagrangian

$$\mathcal{L} = -\tfrac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a \tag{3.1}$$

and the corresponding equations of motion

$$D^\nu F_{\mu\nu} = 0 \tag{3.2}$$

reduce to (2.2) and (2.4) for x_4-independent configurations in the BPS limit $\lambda=0$.

We are left with the problem of finding x_4-independent solutions to (3.2). Because of the Bianchi identities

$$D^\nu F^*_{\mu\nu} = 0, \; F^*_{\mu\nu} := \tfrac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}, \tag{3.3}$$

which hold by virtue of the definition (2.3a), we can simplify our task even further. If we are able to find a solution to the self-duality conditions

$$F_{\mu\nu} = \pm F^*_{\mu\nu}, \tag{3.4}$$

we are guaranteed a solution to the equations of motion (3.2) and therefore to (2.4) in the BPS limit. With our identification $A_4=\Phi$, the self-duality conditions (3.4) are the Bogomol'nyi equations[6)]

$$B_i := \tfrac{1}{2}\epsilon_{ijk}F^{jk} = \pm D_i\Phi. \tag{3.5}$$

The identity

$$E = 4\pi|n| + \tfrac{1}{2}\int d^3x[B^i_a \pm (D^i\phi)_a]^2 \qquad (3.6)$$

shows that solutions to the Bogomol'nyi equations minimize the energy in a topological sector and are therefore stable.

The next step is to find a linear system associated to (3.4), i.e., we are looking for linear equations whose consistency conditions are (3.4). This is achieved by the linear system

$$(A_{\dot\alpha 1} - \xi A_{\dot\alpha 2})k = i(\partial_{\dot\alpha 1} - \xi\partial_{\dot\alpha 2})k =: iD_{\dot\alpha}\, k, \quad \dot\alpha,\alpha = 1,2, \qquad (3.7)$$

in complexified coordinates $x_\mu \in \mathbb{C}$ with

$$x = x_\mu q_\mu, \quad q = (i\vec\sigma, 1), \qquad (3.8)$$

and

$$A_{\dot\alpha\alpha}dx^{\dot\alpha\alpha} = A_\mu dx^\mu, \qquad (3.9)$$

$$k \in GL(2,C), \ \det k = 1. \qquad (3.10)$$

The compatibility condition for (3.7) is exactly the self-duality condition (3.4) in complexified coordinates (3.8). This guarantees that for any $k(\xi,x)$ with $(D_{\dot\alpha}\, k)k^{-1}$ linear in ξ, the $A_{\dot\alpha\alpha}$ defined in (3.7) are automatically self-dual. Implicitly, we have found solutions.

To construct solutions, we must pick k's for which $(D_{\dot\alpha}\, k)k^{-1}$ is linear in ξ and can be calculated explicitly. Furthermore, we are only interested in solutions with the following properties: (i) $A_\mu \in SU(2)$ in some gauge, (ii) A_μ x_4-independent in some gauge, (iii) $|A_4| = |\phi| \to 1$ for $r \to \infty$ with arbitrary winding number $n \in \mathbb{Z}$ (iv) A_μ smooth, C^∞ say. We will discuss the explicit construction and the conditions (i) and (ii) and refer to the literature[8] as far as the conditions (iii) and (iv) are concerned.

The explicit construction of $A_{\dot\alpha\alpha}$ is possible if we find a $G(\xi,x)$ and $k_\pm$ holomorphic in $C \cup C_\pm$ with

$$\det G=1, \ D_{\dot\alpha}\, G=0, \ k_-G=k_+, \ \xi \in C. \qquad (3.11)$$

Here C is an annulus in the ξ-plane and $C_+(C_-)$ is the inside (outside) of C. Now

$$(D_{\dot\alpha}\, k_+)k_+^{-1} = (D_{\dot\alpha}\, k_-)k_-^{-1}, \ \xi \in C, \qquad (3.12)$$

holds, which shows that both sides of this equation are linear in ξ. Furthermore,

$$\begin{aligned} A_{\dot\alpha' 1} &= i(\partial_{\dot\alpha 1}\, k_+^o)(k_+^o)^{-1}, \quad k_+^o := k_+(\xi=0), \\ A_{\dot\alpha' 2} &= i(\partial_{\dot\alpha 2}\, k_-^o)(k_-^o)^{-1}, \quad k_-^o := k_-(\xi=\infty). \end{aligned} \qquad (3.13)$$

We have again shifted the problem. Our task now is to guess the right G, which is called a transition matrix. This task is considerably simplified by casting the Prasad-Sommerfield monopole[7] with n=±1, which had been found by a different method, into the above language. For the Prasad-Sommerfield monopole, the transition matrix G reads

$$\begin{aligned} G &= \begin{pmatrix} \xi & \rho^{(1)} \\ 0 & \xi^{-1} \end{pmatrix}, \quad \rho^{(1)} = e^{u+v}\,\frac{e^\gamma - e^{-\gamma}}{\gamma}, \\ \gamma &= u-v, \ u=i(x_{21}\xi+x_{22}), \ v=i(x_{11}+x_{12}\xi^{-1}). \end{aligned} \qquad (3.14)$$

Because G is triangular it is easy to write out the condition $k_+G=k_-$ and to calculate $k_\pm^o$ explicitly, which, of course, leads back to the Prasad-Sommerfield potentials.

The time-independence of the Prasad-Sommerfield solution and the reality condition $A_\mu \in SU(2)$ can be checked on the level of the transition matrix G. To this end we transform G to

$$\tilde{G} = \Delta_- G \Delta_+ \tag{3.15}$$

with

$$\Delta_- = \begin{pmatrix} e^{-v} & 0 \\ 0 & e^{v} \end{pmatrix} , \quad \Delta_+ = \begin{pmatrix} 0 & -e^{u} \\ e^{-u} & \xi\gamma e^{-u} \end{pmatrix} . \tag{3.16}$$

Because $\Delta_\pm$ is holomorphic in $C \cup C_\pm$, $\tilde{G}$ can be split using $\tilde{k}_- = k_- \Delta_-^{-1}$ and $\tilde{k}_+ = k_+ \Delta_+$, which leads to the same potentials because of $D_{\dot{\alpha}} \Delta_\pm = 0$. The G equivalent to (3.14) reads

$$\tilde{G} = \begin{pmatrix} \frac{e^{\gamma} - e^{-\gamma}}{\gamma} & -\xi e^{-\gamma} \\ \xi^{-1} e^{-\gamma} & \gamma e^{-\gamma} \end{pmatrix} . \tag{3.17}$$

$\tilde{G}$ is independent of x_4 because γ is independent of x_4 and satisfies the condition

$$\tilde{G}(\xi, u, v) = \tilde{G}^+(-\bar{\xi}^{-1}, -\bar{v}, -\bar{u}), \tag{3.18}$$

which can be shown to guarantee $A_\mu \in SU(2)$. The conditions (iii) and (iv) are more difficult to check.

After 't Hooft[9] and Polyakov[10] independently had found the $n=\pm 1$ monopole and Prasad and Sommerfield[7] had explicitly constructed the corresponding solution in the BPS limit, it took five years before Ward[11], whose approach we have sketched here, and, independently using a different method, Forgács, Horvath and Palla[12] were able to write down the corresponding 2-pole solution. Its transition matrix reads

$$\tilde{G} = \begin{pmatrix} \frac{e^{\gamma} - e^{-\gamma}}{\gamma^2 + \pi^2/4} & \xi^2 e^{-\gamma} \\ \xi^{-2} e^{-\gamma} & (\gamma^2 + \pi/4) e^{-\gamma} \end{pmatrix} . \tag{3.19}$$

After this breakthrough everything fell into place. The transition matrix (3.19) was generalized to

$$\tilde{G} = \begin{pmatrix} \frac{e^{K_{n-1}} + (-1)^n e^{-K_{n-1}}}{H_n} & (-\xi)^n e^{-K_{n-1}} \\ \xi^{-n} e^{-K_{n-1}} & H_n e^{-K_{n-1}} \end{pmatrix} . \tag{3.20}$$

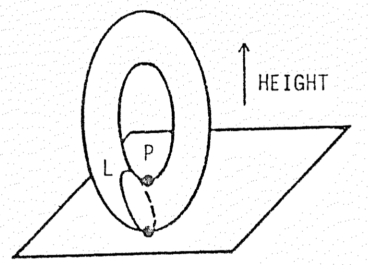
With appropriate conditions on the coefficients of K_{n-1} and H_n, which are polynomials in γ, monopoles with arbitrary winding number and the maximal number of parameters can be identified. For n=2, in principle the constraint equations have been solved[13] and the potentials and the Higgs field can be constructed explicitly, although nobody has done so. We do not have time to discuss either Ward's construction in detail or the different approaches to the monopole problem[5)12)14)].

IV. Saddle points (Sphalerons)

4.1. Noncontractible loops

We now ask the question whether the minima in the topological sectors are the only smooth static finite-energy solutions. The answer will be "no". In fact, we will show that there exist smooth time-independent finite-energy saddle points. To find these we cannot use the Bogomol'nyi equations (3.5) or the associated linear system (3.7). We can, however, use topological methods. It will turn out that in this case not the nontrivial topology of the set of finite-energy configurations itself is relevant but the nontrivial topology of the space of loops of finite-energy configurations.

The underlying idea can be best explained with a simple finite-dimensional example: Let us consider the torus $S^1 \times S^1$ and the function H(x) which for each point x on the torus is given by the height of this point with respect to the plane on which the



torus sits. The question we want to answer is whether the function H has a saddle point. The answer is positive: P is a saddle point, and a way to prove this is the following: For our problem, there exist noncontractible loops (NCL) L and the minimum of H with respect to all NCL of the maxima on each loop is a saddle point. This way of finding a saddle point is known as Morse theory or Ljusternik-Snirelman technique or min-max procedure.

If we want to apply this idea to Yang-Mills-Higgs systems we must replace the manifold $S^1 x S^1$ by the manifold of finite-energy Yang-Mills-Higgs configurations and the function H by the energy functional $E[A_i,\Phi]$. Taubes[15] has shown that the standard Morse theory does not apply to this situation because some of the necessary conditions do not hold. Nevertheless he was able to give a weaker analog of Morse theory and establish the existence of an infinite number of finite-energy saddle points in each monopole sector of SU(2) Yang-Mills-Higgs theory with a Higgs triplet in the adjoint representation in the BPS limit, which is case (B) above.

Here, we will be content with less complicated examples. The toy model we will study first is Φ^4 theory[16]. We will then discuss (C) SU(2) gauge theory with a complex Higgs doublet in 3 space dimensions (G=SU(2), ℓ=4, D=3). Case (C) is the SU(2) part of the bosonic Weinberg-Salam model and therefore an interesting and potentially relevant example.

The Φ^4 theory we want to study is that for a complex field in 1 space dimension with energy functional

$$E = \int_{-\infty}^{\infty} dx\{\tfrac{1}{2}(\partial_x\Phi)^*(\partial_x\Phi) + \frac{\lambda}{4}(|\Phi|^2-1)^2\}. \tag{4.1}$$

Under the necessary smoothness conditions

$$\lim_{x\to-\infty} \Phi(x) = \Phi^- = e^{i\Theta} \tag{4.2}$$

holds for finite-energy configurations. A loop of these configurations, i.e., a family of Higgs fields parametrized by $\mu(0\le\mu\le2\pi)$ with $\Phi(\mu=0) = \Phi(\mu=2\pi) = 1$, now obviously defines a map

$$\Phi^- : S^1 \to S^1.$$

Therefore, the space of loops is topologically nontrivial. Furthermore, we can easily write down noncontractible loops in the form

$$\Phi(\mu,x) = f(x)e^{i\mu}+1-f(x) \tag{4.3}$$

with $f\to1$ for $x\to-\infty$, where different loops are given by different f's.

Our next step is to maximize the energy on each loop. Because

$$E(\mu) = 2\int dx\{f'^2\sin^2\frac{\mu}{2} + 2\lambda f^2(1-f)^2 \sin^4\frac{\mu}{2}\} \tag{4.4}$$

holds for (4.3), we are led to $\mu=\pi$ and the maximal configuration on the loop

$$\Phi = 1-2f(x) \in \mathbb{R}. \tag{4.5}$$

To exclude the trivial configuration we exclude $f\to1$ for $x\to+\infty$ which results in $f\to0$ for $x\to+\infty$ by the finite-energy condition.

Before we minimize the energy for (4.5) we check that a stationary point with respect to variations within the ansatz (4.5) is a stationary point of the full energy functional (4.1). We can do this explicitly by proving that the variational equations of (4.4) with $\mu=\pi$ are the equations of motion of (4.1) for the ansatz (4.5). Minimization now guarantees a solution and the minimum of the energy for the real ansatz (4.5) is, of course, attained by the kink solution

$$\Phi = \tanh\sqrt{\frac{\lambda}{2}}(x-x_0), \tag{4.6}$$

which is stable in a real Φ^4 theory, a saddle point however in a complex Φ^4 theory.

For case (C) we proceed analogously. Because a complex Higgs doublet breaks the SU(2) symmetry completely we must study $\pi_2(G/H) = \pi_2(SU(2))$ if we are interested in finite-energy configurations, and $\pi_3(SU(2))$ if we are interested in loops. This shows that there are noncontractible loops and we follow Manton[17)] to construct some: For a nontrivial map from S^3 to S^3 the obvious ansatz is

$$\phi^\infty_{Re} = \omega(\mu)\begin{pmatrix}\sin\mu\sin\theta\cos\phi\\ \sin\mu\sin\theta\sin\phi\\ \sin\mu\cos\theta\\ \cos\mu\end{pmatrix}, \quad \phi^\infty = \begin{vmatrix}(\phi^\infty_{Re})_1 + i(\phi^\infty_{Re})_2\\ (\phi^\infty_{Re})_3 + i(\phi^\infty_{Re})_4\end{vmatrix} \tag{4.7}$$

$(0 \leq \mu \leq \pi)$. With

$$\omega = \begin{pmatrix}1 & & & \\ & 1 & & \\ & & \cos\mu & -\sin\mu\\ & & \sin\mu & \cos\mu\end{pmatrix} \tag{4.8}$$

we have achieved

$$\phi^\infty(\theta=0) = \begin{vmatrix}0\\ i\end{vmatrix} \tag{4.9}$$

and have defined a loop ϕ^∞.

To find the corresponding A^∞_i, we write

$$\phi^\infty = U^\infty\binom{0}{1}, \quad U^\infty \in SU(2) \tag{4.10}$$

and solve

$$\partial_i\phi^\infty + iA^\infty_i\phi^\infty = 0 \tag{4.11}$$

for A_i:

$$A^\infty_i = i(\partial_i U^\infty)(U^\infty)^+. \tag{4.12}$$

The noncontractible loops are now

$$\phi = h(r)\phi^\infty + [1-h(r)]\begin{vmatrix}0\\ ie^{i\mu}\cos\mu\end{vmatrix}, \tag{4.13a}$$

$$A_i = f(r)A^\infty_i \tag{4.13b}$$

with $h \to 1$ and $f \to 1$ for $r \to \infty$. Here, we have used all the gauge freedom to achieve (4.9) and $x_i A_i = 0$. Given the noncontractible loops we will use the min-max procedure to establish the existence of a saddle point in the next section.

4.2. A saddle point in the SU(2) part of the bosonic Salam-Weinberg model

For our toy model we went through the following procedure: We maximized the energy on the loops, checked the consistency of the resulting ansatz, and minimized the energy for the ansatz. If the minimum is obtained, we have found a solution. We want to repeat these steps here for the loops (4.13).

First, we calculate the energy. This yields

$$\begin{aligned}E = 4\pi\int_0^\infty dr\{&4f'^2\sin^2\mu + \tfrac{1}{2}r^2h'^2\sin^2\mu\\ &+\frac{8}{r^2}f^2(1-f)^2\sin^4\mu + \tfrac{1}{4}\lambda r^2(h^2-1)^2\sin^4\mu\\ &+[h^2(1-f)^2\sin^2\mu - 2fh(1-h)(1-f)\cos^2\mu\ \sin^2\mu\\ &+f^2(1-h)^2\cos^2\mu\ \sin^2\mu]\}.\end{aligned} \tag{4.14}$$

We find that if the conditions

$$0 \leq f,\ h \leq 1, \tag{4.15a}$$

$$(1+\sqrt{2})h(1-f) \geq f(1-h) \tag{4.15b}$$

hold, then $\mu=\pi/2$ is a maximum. The corresponding configurations are

$$\Phi = (H+1)\Omega\binom{0}{1}, \quad \Omega = i\hat{x}\vec{\sigma}\,, \tag{4.16}$$
$$A_i = -i\,\frac{2-F(r)}{2}\,(\partial_i\Omega)\Omega^+,$$

which is the ansatz written down by Dashen, Hasslacher and Neven[18] long ago and later independently by Boguta[19], both before NCL in this model were studied. To make the existence of a saddle point less mysterious we have discussed the underlying topology.

Next we have to check that the variation within the ansatz (4.16) leads to the full Euler-Lagrange equations for the ansatz (4.16). This can be shown explicitly. Furthermore, it can be shown that the ansatz (4.16) supplemented by $a_i=0$ for a further U(1) potential is not compatible with the U(2) gauge theory, which is the full bosonic Salam-Weinberg model. We will come back to this point later.

For SU(2) gauge theory, we can go on and establish the existence of a solution by proving that the energy for the ansatz (4.16)

$$E = 4\pi\int_0^\infty dr\{F'^2 + \tfrac{1}{2}r^2H'^2 + \frac{1}{2r^2}F^2(F-2)^2 + \tfrac{1}{4}F^2(H+1)^2 + \frac{\lambda}{4}r^2H(H+2)\} \tag{4.17}$$

attains its minimum[20]. That this is likely to happen can be seen by inspection of (4.17). In fact, the submodel (4.17) is topologically nontrivial because the finite-energy condition (with the necessary smoothness assumptions) leads to the boundary conditions

$$0,2 \xleftarrow[r\to 0]{} F \xrightarrow[r\to\infty]{} 0. \tag{4.18}$$

The boundary conditions (4.18) show that there are two inequivalent classes, and the minimum in the nontrivial class is our candidate solution.

For a rigorous proof, we use the Tyupkin-Fateev-Shvarts method[21], whose essential steps are the following: First, we consider a minimizing sequence (F_n,H_n),

$$\lim_{n\to\infty} E[F_n,H_n] = \inf E, \tag{4.19}$$

and prove that

$$\|(F_n,H_n)\| = [\int_0^\infty dr(F_n'^2+r^2H_n'^2) + F_n^2(1) + H_n^2(1)]^{\frac{1}{2}} < \infty \tag{4.20}$$

holds. Second, we show that the Hilbert space defined by (4.20) has the Bolzano-Weierstrass property, i.e., each sequence in this space has a weakly converging subsequence. This defines a weak limit (F_0,H_0). Third, we show that the energy functional has the property of sequential weak lower semicontinuity, i.e.,

$$E[F_0,H_0] \leq \lim_{n\to\infty} E[F_n,H_n] \tag{4.21}$$

holds. However, because the r.h.s. of (4.21) is inf E, the equality sign holds and (F_0,H_0) attains the minimum. We have proved the existence of a solution.

There are, however, still a few points to be clarified:
(i) Because we considered the nontrivial class we chose $F_n\to 2$ $(r\to 0)$ for our minimizing sequence. We must show that $F_0\to 2$ $(r\to 0)$, i.e. that we have not proved the existence of the trivial solution with E=0. This is ruled out by the inequality

$$|F_n(r)-2| \leq (\int_0^r ds F_n'^2(s)\int_0^r ds)^{\frac{1}{2}} \leq c\sqrt{r} \tag{4.22}$$

with an n-independent constant c. It shows that for sufficiently small but nonzero r,F is already arbitrary close to 2. (ii) We must prove regularity which is a rather technical proof and will be omitted here[20]. Finally, (iii) We must reconsider the question of instability because the inequalities (4.15) are not guaranteed to hold for the solution (f_0,h_0). Instead of trying to prove (4.15) for the solution we simply generalize the ansatz (4.16) to

$$\Omega = \cos\theta + i\hat{x}\vec{\sigma}\sin\theta. \tag{4.23}$$

Now, (4.16) is the special case $\Omega=\pi/2$ of the family of ansätze (4.23). The corresponding energy is

$$E = 4\pi\int_0^\infty dr(F'^2\sin^2\theta + \tfrac{1}{2}r^2H'^2 + \alpha^2\sin^2\theta + 2\beta^2\sin^4\theta + \gamma^2), \tag{4.24}$$

which implies

$$E(F_{\pi/2}, H_{\pi/2}, \theta < \pi/2) < E(F_{\pi/2}, H_{\pi/2}, \theta = \pi/2). \tag{4.25}$$

Our solution is a saddle point.

We know already that we cannot embed our SU(2) solution into the bosonic Salam-Weinberg model. In fact, for this solution the U(1) current

$$J_i \sim g'[\phi^+ D_i\phi - (D_i\phi)^+\phi] \tag{4.26}$$

is nonzero and we cannot satisfy the U(1) equation $\partial^j f_{ij} = J_i$ of the U(2) model by putting $a_i=0$. Klinkhamer and Manton [22], however, adopt the attitude that the nontrivial topology of the loop space is reason enough to expect the existence of a solution and that the SU(2) solution should be a good approximation for $g' \ll 1$. They therefore calculate the first order correction in g' to the SU(2) solution and the energy of the solution with a result of approximately 10 TeV. They find that in this approximation the U(2) configuration has an electric dipole field with magnetic moment $\mu = 0.216\ \mathrm{GeV}^{-1} (\lambda=0)$.

This concludes our discussion of the classical solution itself. What is left is a discussion of its relevance. Kuzmin, Rubakov and Shaposhnikov[23] have made some contribution to this discussion together with some calculations of the baryon-number nonconservation by electroweak processes. They argue that, whereas at T=0 instantons describe the tunnelling between topologically inequivalent vacua, for higher temperature the system can pass over the barrier between the different vacua and that the dominant contribution to the rate of the vacuum decay comes from the saddle point. The aim of their calculation is to estimate the generation of baryon-asymmetry of the universe by electroweak processes. The result is negative for a second-order phase transition and nonconclusive for a first-order phase transition, and more work has to be done to give the final answer to the question of the relevance of saddle points.

V. Remarks on the scattering of slowly-moving monopoles

Before we turn to the discussion of existence proofs for time-dependent solutions of the exact equations of motion I would like to make a few remarks on some exciting new results describing slowly-moving monopoles. These results are based on an idea by Manton[24]. Manton argues that the scattering of slowly-moving BPS monopoles is controlled by a geodesic motion in the parameter space of the static multi-monopole solution we have discussed before.

To find these geodesics Manton suggested to determine A_0 first by solving Gauss's law

$$D_i E_i = [D_0\Phi, \Phi] \tag{5.1}$$

for the n-pole configuration $(A_i(s(t)), \Phi(s(t)))$ with 4n time-dependent parameters s^k. (For time-independent s this configuration is the static solution discussed above.) The next step is to read off the metric $g_{k\ell}$ from the kinetic energy term

$$E_{kin} = \tfrac{1}{2}\int d^3x [E_i^a E_i^a + (D_0\Phi)^a (D_0\Phi)^a] \simeq g_{ke}\dot{s}^k \dot{s}^\ell. \tag{5.2}$$

Given the metric the final task is to calculate the geodesics.

Because a lot has been learned about the parameter space since Manton put forward his idea, Atiyah and Hitchin[25] were able to carry out his programme without following literally each step. Using general results and symmetry considerations they were able to write down the metric for the 2-pole solution and find some interesting geodesics. To interpret these geodesics one has to know that 3n of the 4n parameters are the coordinates whose time-derivatives are the momenta, and that n of the 4n parameters are phase angles whose time-derivatives are the electric charges. With this interpretation one sees that monopoles can be converted into dyons, which is the most exciting result Atiyah and Hitchin found.

VI. Global existence proofs

6.1. Segal's theorem

We go back to the discussion of the full Yang-Mills-Higgs equations without any approximation. This is, of course, a difficult problem, and we have to be content with some mathematical results and cannot expect to make easily contact with the results of the above treatment of monopole-monopole scattering. The result we will derive is that of global existence of solutions which should act as an underpinning of all approximation techniques. We will show that for initial value data from certain spaces solutions exist and do not develop singularities. That this is not a physicaly irrelevant statement can easily be seen by comparing this result to the situation in general relativity. There, a global existence proof does not exist. In fact, it is known that regular initial value data can develop a black hole singularity in a finite time.

Our existence proof is based on Segal's theorem[26)], which we are going to state, discuss and illustrate now.

Theorem: Let $W(s,t)$ be a function from ordered pairs $(s \geq t)$ in $T=[0,\infty)$ to linear, continuous transformations on a Banach space B, such that

$$W(t'',t')W(t',t) = W(t'',t),\ W(t,t) = I, \tag{6.1}$$

for $t \leq t' \leq t''$. For each $t \in T$, let K_t be an operator on B which, uniformly on each finite interval in T, is locally lipschitzian:

$$\bigwedge_{t',c}\ \bigwedge_{\substack{u,v\in B\\ \|u\|,\|v\|<c\\ t\in[0,t']}}\ \bigvee_{f(c,t')} \|K_t(u)-K_t(v)\| \leq f(c,t)\|u-v\|, \tag{6.2}$$

and such that $K_t(u)$ is a continuous function of $(t,u) \in T\times B$.

Then for any given element u_0 of B, the maximum interval $[0,\bar{t})$ of existence of the necessarily unique continuous function u from such an interval to B such that

$$u(t) = W(t,0)u_0 + \int_0^t W(t,s)K_s(u(s))ds \tag{6.3}$$

has positive length, and is either all of T, or else

$$\|u(t)\| \to \infty \quad \text{as} \quad t \to \bar{t}. \tag{6.4}$$

In his proof, Segal considers the sequence

$$u_{n+1}(t) = W(t,0)u_0 + \int_0^t W(t,s)K_s(u_n(s))ds. \tag{6.5}$$

He shows by induction that u_{n+1} is continuous and that u_{n+1} does not

blow up locally, if u_0 is continuous and its norm is bounded. This shows that u_n is a sequence in B. Using the Lipschitz condtion (6.2) he can then show that u_n is a Cauchy sequence. Because B is a Banach space the limit of the sequence u_n exists, and it can be shown that it satisfies the equation (6.3). This is the local result. To establish the global result assume that (6.4) does not hold. Then, by repeating the above arguments for $\bar{t}$, we can extend the interval of existence to $[0,\bar{t}+\epsilon)$. If the norm never blows up we can extend the interval of existence to $[0,\infty)$.

So far we have only discussed the integral equation (6.3). Segal[26)] also addresses himself to the corresponding differential equation

$$\frac{d}{dt}u(t) = Au(t) + K_t(u(t)), \tag{6.6}$$

where A is defined by

$$\lim_{\epsilon\to 0} \frac{1}{\epsilon}[W(t+\epsilon,t)-I]y = Ay,\ y\in D_A. \tag{6.7}$$

He shows that $u \in D_A$ for $t \in [0,\bar{t})$ and that u satisfies the differential equation (6.6) if the following conditions hold: (i) A generates a 1-parameter semigroup on B. (ii) $K_t(u)$ is C^1. (iii) u satisfies the integral equation (6.3). (iv) $u(t_0) \in D_A$.

Let us illustrate Segal's theorem by applying it to the simple example of ϕ^4 theory. The equation of motion is

$$\partial_t^2\phi = \partial_x^2\phi - \phi^3,\ \phi(t,x)\in \mathbb{R}. \tag{6.8}$$

The corresponding integral equation reads

$$\psi(t) = e^{At}\psi(0) + \int_0^t ds\, e^{A(t-s)}K(\psi(s)) \tag{6.9}$$

with

$$\psi = \begin{vmatrix} \phi \\ \pi=\partial_t\phi \end{vmatrix},\ A = \begin{vmatrix} 0 & 1 \\ \partial_x^2-m^2 & 0 \end{vmatrix},\ K = \begin{vmatrix} 0 \\ m^2\phi-\phi^3 \end{vmatrix}. \tag{6.10}$$

For the linear problem (K=0) alone we have the result that A generates a 1-parameter semigroup on each Sobolev space $H_{s+1}\times H_s$ with $s \geq 0$, where

$$H_s = \{f \quad D \,|\, \|f\|^2 = \|f\|^2_{L^2} + \dots + \|\underbrace{\partial\dots\partial}_{s} f\|^2_{L^2} < \infty\}. \tag{6.11}$$

To treat the nonlinear part K in this and in the following examples we will make repeated use of the Gagliardo-Nirenberg inequalities

$$\|f\|_{L^p} \leq C\|\underbrace{\partial\dots\partial}_{m} f\|^a_{L^r}\|f\|^{1-a}_{L^q}, \quad f: R^n \to R^k,$$
$$\frac{1}{p} = a[\frac{1}{r} - \frac{m}{n}] + (1-a)\frac{1}{q}, \quad 0 \leq a \leq 1. \tag{6.12}$$

(6.12) implies, in particular,

$$\|f\|_{L^6} \leq C\|f\|_{H_1}, \quad \|f\|_{L^8} \leq C\|f\|_{H_1}, \tag{6.13}$$

which allows us to check two of the essential conditions of Segal's theorem: First, K is an operator on $B=H_1xL^2$ because $\|\phi\|_{H_1} < \infty$ implies

$$\|m^2\phi - \phi^3\|_{L^2} \leq C_1\|\phi\|_{L^2} + \|\phi\|^3_{L^6} < \infty. \tag{6.14}$$

Second, the Lipschitz condition holds because we can derive

$$\|m^2(\phi_1-\phi_2) - \phi_1^3 + \phi_2^3\|_{L^2}$$
$$\leq C_1\|\phi_1 - \phi_2\|_{H_1} + \|\phi_1^2 + \phi_1\phi_2 + \phi_2^2\|_{L^4}\|\phi_1 - \phi_2\|_{L^4} \tag{6.15}$$
$$\leq [C_1 + C_2(\|\phi_1\|^2_{L^8} + \|\phi_2\|^2_{L^8})]\|\psi_1 - \psi_2\|.$$

This essentially completes the local existence proof.

For the global existence proof we must show that the H_1xL^2 - norm does not blow up in a finite time. To this end, we consider a local solution of the differential equation from the domain of A, $D_A=H_2xH_1$, and define the pseudo-energy

$$\overline{E} = \int_{-\infty}^{\infty}dx\{\tfrac{1}{2}\pi^2+\tfrac{1}{2}(\partial_x\phi)^2 + \tfrac{1}{4}\phi^4+\tfrac{1}{2}\phi^2\} < \infty . \tag{6.16}$$

Locally, we can now use the equations of motion to show

$$\frac{d}{dt}\overline{E} = \int_{-\infty}^{\infty}dx\phi\pi \leq C\sqrt{\overline{E}}, \tag{6.17}$$

which implies

$$\overline{E} \leq (C_0+\tfrac{1}{2}Ct)^2. \tag{6.18}$$

This shows that $\overline{E}$, and therefore the H_1xL^2 - norm of ψ, does not blow up, which is the global existence result.

Things get slightly more complicated in the case of symmetry breaking:

$$\partial_t^2\phi = \partial_x^2\phi + \phi(1-\phi^2). \tag{6.19}$$

In this case, the finite-energy condition makes the configurations with $\phi^2 \to 1$ for $x \to \pm\infty$ the interesting ones. However, these configurations are not square-integrable. To remedy the situation we can subtract a background field $\overset{o}{\phi}$:

$$\phi = \overset{o}{\phi} + \phi, \; \partial_t\overset{o}{\phi} = 0, \tag{6.20}$$

and work with the subtracted fields ϕ and $\pi=\partial_t\phi$. If we now impose conditions on $\overset{o}{\phi}$, in this case,

$$\sup_{x\in R} |\overset{o}{\phi}(x)| < \infty, \; \partial_x\overset{o}{\phi} \in H_1, \; 1-\phi^2 \in L^2 \tag{6.21}$$

we can repeat our local and global existence proof for (6.19). Of course, we have to show that background fields exist which satisfy all the conditions we need for our proof.

Because we are mainly interested in topologically nontrivial solutions we would also like to keep control over the asymptotic behaviour. This is possible in the above example. There, $\phi \in {}_1$, which implies $\phi \to 0$ for $x \to \pm\infty$ and therefore

$$\lim_{x\to\pm\infty} \phi(t,x) = \lim_{x\to\pm\infty} \overset{o}{\phi}(x). \tag{6.22}$$

The topology does not change.

6.2. Time dependent vortices and monopoles

The general theory we have discussed above has been applied to gauge theories by many authors, including Segal[27] himself. As far as the global existence proof is concerned, the main contributions have been made by Moncrief[28] for the Maxwell-Klein-Gordon field in 2+1 dimensions, and by Eardley and Moncrief[29] for Yang-Mills-Higgs fields in 3+1 dimensions. The treatment of these authors, however, excludes topologically nontrivial fields, which are those we are particularly interested in. Using the background field method, the general techniques have therefore been adopted to treat vortices[30] and monopoles[31]. Here, we will discuss the main points of the proofs without going into the many technical details.

For the Landau-Ginzburg theory (A), we subtract the background fields $\mathring{A}_\mu, \mathring{\phi}$:

$$A_\mu = \mathring{A}_\mu + a_\mu, \quad \Phi = \mathring{\phi} + \phi, \tag{6.23}$$

and impose the following conditions on them:

$$\mathring{A}_0 = \partial_t \mathring{A}_i = \partial_t \mathring{\phi} = 0, \quad \partial_i \mathring{A}_i = 0,$$

$$\sup_{\vec{x} \in \mathbb{R}^2} |\underbrace{\partial_i \partial_j \ldots}_{m} \mathring{\phi}| < \infty, \quad \sup_{\vec{x} \in \mathbb{R}^2} |\underbrace{\partial_j \partial_k \ldots}_{m-1} \mathring{A}_i| < \infty, \quad m=0,1,2, \tag{6.24}$$

$$|\mathring{\phi}|^2 - 1 \in L^2, \quad \mathring{\nabla}_i \mathring{\phi} := \partial_i \mathring{\phi} + i \mathring{A}_i \mathring{\phi} \in H_2, \quad \mathring{F}_{ij} \in H_2, \quad \partial_i^2 \mathring{A}_j \in H_2.$$

It can be shown that background fields which satisfy these conditions exist. In fact, the known rotationally symmetric static vortex solutions[32] satisfy all of these conditions.

We now work with the subtracted fields

$$\psi^T = (a_0, P_0, a_1, P_1, a_2, P_2, \phi, \pi),$$
$$P_\mu := \partial_t a_\mu, \quad \pi := \partial_t \phi + i a_0 \phi, \tag{6.25}$$

and base our choice of Banach space on the results for the corresponding linear problem given by the operator A of the Landau-Ginzburg model. Because A generates a 1-parameter semigroup on each Sobolev space $H^{(s)} := (H_{s+1} \times H_s)^4$ for $s \geq 0$, we attempt to work in $H^{(0)}$ first. This is not possible, as can be seen by looking at the component $K_7 = -i a_0 \phi$. If $a_0, \phi \in H_1$ we cannot conclude $K_7 \in H_1$. However, if $f \in H_2$, then the inequalities (6.12) guarantee $\|f\|_{L^\infty} < \infty$. Therefore, $a_0, \phi \in H_2$ implies $K_7 \in H_2$. We can work in $H^{(1)}$, which because of the higher order, will complicate the global existence proof considerably.

Another problem we have to face is a result of the gauge freedom. If we impose the Lorentz condition $\partial_\mu a^\mu = 0$, as we do, the fields must satisfy the constraint

$$\Delta a_0 - \partial_t \partial_i a_i = i[(\mathring{\phi} + \phi)(\pi^* - i a_0 \bar{\mathring{\phi}}) - (\bar{\mathring{\phi}} + \bar{\phi})(\pi + i a_0 \bar{\mathring{\phi}})], \tag{6.26}$$

so, not only do we have to prove that a solution exists but we also must prove that the constraint (6.26) is propagated. Both can be done locally in $H^{(1)}$ for the integral equation and in $H^{(2)}$ for the corresponding differential equation.

For the global existence proof we must show that the $H^{(1)}$ - norm of ψ does not blow up. For this, it is not enough to use energy conservation alone. In addition, we must show that the higher order pseudo-energy

$$E_2 = \int d^2x \{ \tfrac{1}{2}(\partial_i E_j)^2 + \tfrac{1}{4}(\partial_i F_{jk})^2 + |D_i D_0 \phi|^2$$
$$+ |D_i D_j \phi|^2 + \tfrac{\lambda}{4}(\bar{\phi} D_i \phi + \phi \overline{D_i \phi})^2 \} \tag{6.27}$$

does not blow up. By repeated use of the *Gagliardo-Nirenberg* inequalities (6.12) one can show easiest in Coulomb gauge that for a local solution the gauge invariant quantity (6.27) satisfies

$$E_2(t) \leq E_2(0) + C_o t + \tfrac{1}{3}C_1 t^3 + \int_0^t ds\, C_2[E_2(0) + C_o s + \tfrac{1}{3}C_1 s^3]e^{C_2(t-s)} . \tag{6.28}$$

This guarantees that E_2 does not blow up, and some further estimates show that the $H^{(1)}$ - norm of ψ does not blow up either. This completes the global existence proof. What is left is to add the result for the winding number n: n(t) is defined for all t and equal to n(0) given by the background field.

In the monopole case[31], additional difficulties have to be overcome. Again, we try to model our proof on the topologically trivial case which in this case is the Eardley-Moncrief proof[29]. However, for a technical reason already the choice of Sobolev space must be different. Eardley and Moncrief work in the $A_o=0$ gauge which implies the constraint

$$\partial_i F_{oi} = [F_{oi}, A_i] - (D_o\phi \cdot T_a\phi)T_a = : \rho . \tag{6.29}$$

They formally solve this constraint:

$$F^c_{oi} = -\frac{1}{4\pi}\partial_i \int d^3x' \frac{\rho(x')}{|x-x'|} , \tag{6.30}$$

and show that for their choice of Sobolev space:

$$(A_i, F_{oi}, \phi, D_o\phi) \in (H_2 x H_1)^2, \tag{6.31}$$

$F^c_{oi} \in H_2$ holds. In the topologically nontrivial case, we cannot prove $F^c_{oi} \in H_2$.

In the topologically nontrivial case, we subtract the background fields $\overset{o}{\phi}$ and $\overset{o}{A}_i$, and put all the following fields into H_2:

$$a_i, \partial_t a_i, b_i : = \epsilon_{ijk}(\partial_j a_k + a_j a_k + [\overset{o}{A}_j, a_k]),$$

$$\phi, \partial_t \phi, \psi_i : = \partial_i \phi + a_i \phi + a_i \overset{o}{\phi} + \overset{o}{A}_i \phi .$$

That means, we work in a higher Sobolev space for which we have to pay in the global existence proof. Furthermore, we introduce additional fields b_i and ψ_i into the equations of motion, which means there are additional constraints whose propagation in time has to be guaranteed. All these problems can be solved and a local existence proof in the spirit of Segal[27] or Ginibre and Velo[33] can be given. Global existence is established by pushing the Eardley-Moncrief technique to the necessary order.

All of these results are very technical. They constitute, however, the first rigorous results concerning time-dependent vortices and monopoles. They may therefore help to decide definitively whether vortices and monopoles are solitons in the sense that they emerge from a collision essentially unchanged.

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