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NEUTRINO MASS EFFECTS
IN NEUTRINO-ELECTRON ELASTIC SCATTERING

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ABSTRACT

A covariant formulation is given for the mass dependent differential cross-sections for neutrino(antineutrino)-electron elastic scattering with massive neutrinos. It is explained how these cross-sections along with a formulation for neutrino oscillations may be used to describe the helicity transformation effect for neutrinos passing through matter.

I. INTRODUCTION

In recent years much effort has been devoted to developing a better understanding of the properties of massive neutrinos[1] and their relation to the gauge theory of electro-weak forces. In this paper, I investigate phenomena which are associeted with the scattering in matter of massive neutrinos from electrons.

An important consequence of the massiveness of neutrinos is the prediction of oscillations which can occur both in vacuum and in matter. Among the oscillations which have been considered are those which change the lepton type(flavor changes)[2], particle-antiparticle oscillations[3], helicity changes and doublet-singlet neutrino changes[4]. Other consequences are also associated with massive neutrinos. These include mass dependent effects in the scattering cross-sections, changes in the direction of the neutrino's spin polarization vector as the result of scattering or as the result of interacting with a strong magnetic field[5].

In this paper, I give invariant cross-sections for the scattering of massive neutrinos from electrons. From these expressions, one may determine the changes in the spin polarization of the neutrino as the result of scattering from electrons in matter. This effect is shown to be enhanced in the presence of neutrino oscillations. An estimate is made for the helicity transformations which can occur for neutrinos passing through matter. It is suggested that scattering induced helicity oscillations may appear in certain astrophysical environments.

II. NEUTRINO-MASS DEPENDENT CROSS-SECTIONS

In this section I present the details of a derivation of the cross-sections for the elastic scattering of a massive neutrino from an electron. In the derivation, I use the units $m_e = \hbar = c = 1$ and the conventions which may be found in Ref. 6. I consider the process in which a neutrino of mass m, polarization four-vector s_a, and four-momentum a is scattered from an electron to a final state of spin polarization s_c and four-momentum c. Initially, the electron is unpolarized and has four-momentum b. Its final state four-momentum is d.

The neutrino and the electron are described respectively by the current four-vectors

$$j_\nu(c,a)^\mu = \bar{\nu}(c)\,\Gamma(1,\lambda)^\mu\,\nu(a) \tag{2.1a}$$

$$j_e(d,b)^\mu = \bar{u}(d)\,\Gamma(V,A)^\mu\,u(b) \tag{2.1b}$$

where

$$\Gamma(V,A)^\mu = \gamma^\mu (V + A\gamma^5)/2 \tag{2.2}$$

In (2.1a) the parameter λ has the value 1 for a right-handed projection and the value -1 for a left-handed projection of the neutrion's helicity. For a Dirac neutrino produced by a V - A process, the value of λ distinguishes between the neutrino and the antineutrino. However, for the case of Majorana neutrinos[7] where $\nu(p) = C\bar{\nu}^T(-p)$ (C represents charge conjugation) the values λ = 1 and -1 represent respectively right- and left-handed projections of the neutrino's helicity.

In the remainder of this section, I will restrict the discussion to that of Dirac neutrinos, and I will assume that these neutrinos are produced from V-A processes so that they are predominately left-handed. The case of Majorana neutrinos can be easily understood with the above mentioned change in the interpretation of the parameter λ.

In the standard model of electro-weak interactions[8], modified so as to include massive neutrinos, the electron neutrino interacts with both the neutral Z and with the charged W vector bosons. The effects for both the neutral and the charged vector bosons may be accounted for with different values for the constants V and A in (2.2). The interaction amplitude for the process under consideration is

$$\mathcal{L}(s_a, s_c) = \frac{g}{\pi\sqrt{2}}\, j_\nu(c,a)\cdot Z(d,b) \tag{2.3}$$

where

$$Z(d,b)^\mu = \frac{4\pi}{(q^2 - M^2)}\left(g^{\mu\nu} - q^\mu q^\nu / M^2\right) j_e(d,b)_\nu \tag{2.4}$$

with q = a - c. The invariant differential cross-section with the amplitude (2.3) for the process $\nu + e \rightarrow \nu + e$ becomes with $4t = (a - c)^2$

$$\frac{d\sigma}{dt}(s_a, s_c, \lambda, m) = \frac{2}{4\pi f(s,a,b)}\, \mathcal{M}(s_a, s_c, \lambda, m) \tag{2.5}$$

where

$$f(s,a,b) = 4((a.b)^2 - m^2)$$

In (2.5), the polarization function is

$$\mathcal{M}(s_a, s_c, \lambda, m) = 8G^2 \,\mathrm{Tr}\left[\rho_c \Gamma(1,\lambda)^\mu \rho_a \bar{\Gamma}^\nu(1,\lambda)\right] \cdot \mathrm{Tr}\left[\rho_d \Gamma(V,A)_\mu \rho_b \bar{\Gamma}(V,A)_\nu\right], \tag{2.6a}$$

with

$$\bar{\Gamma} = \gamma^0 \Gamma^\dagger \gamma^0$$

where ρ_i (i = a,b,c,or d) denotes the polarization density matrices.

Upon evaluation[9], one finds for large M

$$\begin{aligned}
\mathcal{M}(s_a, s_c, \lambda, m) = {} & 2G^2\Big[(a\cdot b)^2 + (a\cdot d)^2 + \\
& m^2\big((s_c\cdot d)(s_a\cdot b) + (s_c\cdot b)(s_a\cdot d)\big) - \\
& \lambda m\big((s_c\cdot d)(a\cdot b) + (s_c\cdot b)(a\cdot d)\big) - \\
& \lambda m\big((a\cdot b)(s_a\cdot b) + (a\cdot d)(s_a\cdot d)\big)\Big]\left(|V|^2 + |A|^2\right) \\
& + 2G^2\Big[-(a\cdot c) - m^2(s_a\cdot s_c) + \\
& \lambda m(s_c\cdot a) + \lambda m(s_a\cdot c)\Big]\left(|V|^2 - |A|^2\right) \\
& - 2G^2\Big[-\lambda\left((a\cdot b)^2 - (a\cdot d)^2\right) - \\
& \lambda m^2\big((s_c\cdot d)(s_a\cdot b) - (s_c\cdot b)(s_a\cdot d)\big) + \\
& m\big((s_c\cdot d)(a\cdot b) - (s_c\cdot b)(a\cdot d)\big) + \\
& m\big((a\cdot b)(s_a\cdot b) - (a\cdot d)(s_a\cdot d)\big)\Big]\left(VA^* + V^*A\right).
\end{aligned} \tag{2.6b}$$

Although information for all scattering configurations with with different values for s_a and s_c can be found from (2.6), a considerable simplification occurs when the neutrino mass is small relative to its energy ($m/\omega_a < 1$). With this approximation, one finds for the scattering of helical neutrinos

$$
\begin{aligned}
\mathcal{M}(\lambda_a,\lambda_c,\lambda,m) \cong\ & 2G^2(1+\lambda_a\lambda_c-\lambda\lambda_a-\lambda\lambda_c)\cdot \\
& [((a\cdot b)^2+(a\cdot d)^2)(|V|^2+|A|^2) - (a\cdot c)(|V|^2-|A|^2)+ \\
& -\lambda((a\cdot b)^2-(a\cdot d)^2)(VA^*+V^*A)] + \\
& m^2 2G^2[(\lambda-\lambda_c)\lambda_a(f_a(b)\,a\cdot b + f_a(d)\,a\cdot d)+ \\
& \quad (\lambda-\lambda_a)\lambda_c(f_c(d)\,a\cdot b + f_c(b)\,a\cdot d)](|V|^2+|A|^2) \\
& - m^2 2G^2[(\lambda-\lambda_c)\lambda_a f_a(c) + (\lambda-\lambda_a)\lambda_c f_c(a)](|V|^2-|A|^2) \\
& - m^2 2G^2[(\lambda\lambda_c-1)\lambda_a(f_a(b)\,a\cdot b - f_a(d)(a\cdot d)) + \\
& \quad (\lambda\lambda_a-1)\lambda_c(f_c(d)a\cdot b - f_c(b)(a\cdot d))](VA^*+V^*A).
\end{aligned}
\tag{2.7}
$$

In this expression, I have used the representation

$$s_p = \lambda_p\left(\frac{|\underset{\sim}{p}|}{m},\ \frac{\omega_p}{m}\underset{\sim}{e}_p,0,0\right) \tag{2.8}$$

for the polarization four-vector of a particle of mass m, energy ω_p and momentum $|\underset{\sim}{p}|\underset{\sim}{e}_p$. I have also defined the scalar products

$$
\begin{aligned}
s_p\cdot q &\cong \frac{\lambda_p}{m}\left(p\cdot q - m^2 f_p(q)\right) \\
s_a\cdot s_c &\cong \frac{\lambda_a\lambda_c}{m^2}\left(a\cdot c - m^2 f_c(a) - m^2 f_a(c)\right)
\end{aligned}
\tag{2.9}
$$

where

$$f_p(q) = \frac{\omega_q}{2\omega_p} + \frac{|\underset{\sim}{q}|}{2|\underset{\sim}{p}|}\underset{\sim}{e}_p\cdot\underset{\sim}{e}_q$$

The scattering of a helical neutrino without a change in helicity is described with the values $\lambda=-\lambda_a-\lambda_c=1$. The corresponding scattering for an antineutrino is described when these parameters have the opposite values. The case of a helicity transformation is described when $\lambda_c=-\lambda_a$. In the standard model the parameters V and A have the values

$$
\begin{aligned}
V &= 1/2 + 2\sin^2\theta_w \\
A &= 1/2
\end{aligned}
\tag{2.10a}
$$

for the scattering of a neutrino and a charged lepton from the same flavor family where both neutral and charged vector bosons contribute. For the scattering of a neutrino and a charged lepton from a different flavor family, when only the neutral vector boson contributes, the parameters have the values

$$V = -1/2 + 2\sin^2\theta_w \qquad (2.10b)$$

$$A = -1/2$$

As a check on the formulae, it is easy to see that the cross-section for the scattering of a massless neutrino can be recovered in the limit $m \to 0$. If one introduces the variable

$$y = b\cdot(a-c)/a\cdot b,$$

then

$$\frac{d\sigma}{dy}(V,A) = -(\omega_a/2)\,\frac{d\sigma}{dt}(V,A) \qquad (2.11a)$$

Upon integrating y from 0 to 1, one finds the total cross-section for elastic electron-neutrino scattering

$$\sigma(V,A) = \frac{\omega_a G^2}{2\pi}\left[\,|V+A|^2 + \frac{|V-A|^2}{3} - \frac{(|V|^2-|A|^2)}{2\,\omega_a}\right] \qquad (2.11b)$$

The result for antineutrino scattering is found from the above expression when $A \to -A$.

III. OSCILLATION EFFECTS

As a massive neutrino passes through matter, its helicity may be reversed as the result of interacting with electrons. As one can see from the previous discussion, this effect has the energy dependence $(m/\omega_a)^2$ and is expected to be small. If on the other hand, the neutrino oscillates to a neutrino type with a larger mass, then the helicity reversal effect can become enhanced. After scattering in the larger mass state, the neutrino can oscillate back to the original or to another lepton type with reversed helicity. For the Dirac neutrino, this combined effect could simulate particle-antiparticle oscillations.

In this section, I give a formulation to estimate the significance of these combined effects. Although it now appears that $\nu_e \longleftrightarrow \nu_\mu$ oscillations have a small probability[10], oscillations of the type $\nu_e \longleftrightarrow \nu_\tau$ may still be significant, especially if the mass of ν_τ is such that $m \sim 1$. At present, this is allowed as the result of the current terrestrial experimental bounds for the neutrino masses.

To begin the discussion, one can use $|\nu_\sigma)$ (σ = 1,2, or 3) to represent eigenstates of the neutrino energy operator with mass eigenvalues m_σ , and $|\nu_\ell)$ (ℓ = e, μ, or τ) to represent the observed physical neutrinos. To describe oscillations, one considers the superposition

$$|\nu_\ell) = U_{\sigma\ell}\,|\nu_\sigma) \tag{3.1}$$

where there is a summation on repeated indices. The time development of this state is generated by the Hamiltonian H so that

$$|\nu_\ell(\tau)) = e^{-iH\tau}|\nu_\ell(0)) \tag{3.2}$$

The probability to observe a neutrino of lepton type ℓ' at time τ , if a neutrino of lepton type ℓ is present at time zero, is

$$W_{\ell\ell'}(\tau) = |a_{\ell'\ell}(\tau)|^2 = U_{\sigma'\ell'}\,U_{\sigma\ell'}\,U_{\sigma'\ell}\,U_{\sigma\ell}\cdot \cos[(E_{\sigma'} - E_\sigma)\tau] \tag{3.3}$$

where

$$\begin{aligned} a_{\ell'\ell}(\tau) &= (\ell'|\ell(\tau)) \\ (\sigma|\ell) &= U_{\sigma\ell} \\ (\ell'|\sigma') &= U_{\sigma'\ell'} \end{aligned} \tag{3.4}$$

If the neutrino energy is large relative to its mass, one finds where the vacuum oscillation length is

$$L_\nu = \frac{4\pi\,|p|^2}{|m_{\sigma'}^2 - m_\sigma^2|} \tag{3.5}$$

As the result of coherent scattering effects, the probabilities (3.3) become modified for oscillations in matter. The appropriate expressions for oscillations in matter can be found in Ref. 11. For either $\nu_e \leftrightarrow \nu_\mu$ or $\nu_e \leftrightarrow \nu_\tau$ oscillations in matter, the transition probabilities become

$$\begin{aligned} |(\nu_e|\nu_\mu(x)|^2 &= 1 - |(\nu_e|\nu_e(x)|^2 \\ &= \tfrac{1}{2}\sin^2(2\theta_\nu)(L_m/L_\nu)^2(1 - \cos(2\pi x/L_m)) \end{aligned} \tag{3.6}$$

Here the oscillation length in matter is

$$L_m = L_\nu\left[1 + \left(\frac{L_\nu}{L_o}\right)^2 - 2\cos 2\theta_\nu\left(\frac{L_\nu}{L_o}\right)\right]^{-1/2} \tag{3.7}$$

and $L_o = 2\pi/GN_e$ where N_e is the electron density.

One can now use the above results along with the cross-sections derived in Section II, to obtain expressions for the passage of neutrinos through matter when they interact with electrons. This provides a description of the helicity transformation effect which is coupled with lepton type oscillations. If one assumes that the flavor of the neutrino is unchanged as the result of the interaction, then the differential cross-section at time τ becomes

$$\frac{d\sigma}{dt}(s_a, s_c, \lambda, \tau_f, \tau, \tau_0)_{\ell_f, \ell'} = \sum_{\ell = e, \mu, \tau} A(\tau_f, \tau, \tau_0)_\ell \frac{d\sigma}{dt}(s_a, s_c, \lambda, m)_\ell \qquad (3.8)$$

where

$$A(\tau_f, \tau, \tau_0) = |a_{\ell\ell_f}(\tau_f - \tau)|^2 \, |a_{\ell'\ell}(\tau - \tau_0)|^2 .$$

In this process, a neutrino of lepton type ℓ' at time τ_0 interacts at time τ as a neutrino of lepton type ℓ with an electron. At time τ_f the neutrino is detected as a neutrino of lepton type ℓ_f. In (3.8) the four-vectors a, c, s_a, and s_c are functions of the value of m at the time of interaction. The probability functions are found form (3.3) when the neutrino is in vacuum and from (3.6) when it is in matter.

If only the final state electron is detected, then one finds upon summing over the final neutrino types and integrating over t as done in deriving (2.11) the expressions for the total cross-section for an interaction at time τ.

$$\sigma(\tau, \lambda_a, \lambda_c, \lambda) = \sum_{\ell = e, \mu, \tau} W_{\ell e}(\tau)\, \sigma_\ell(\lambda_a, \lambda_c, \tau - \tau_0). \qquad (3.9)$$

In the limit where one neglects terms which depend upon m/ω_a, one finds the expressions used in Ref. 12 to study oscillations in ν_e- e scattering. In the same limit, one obtains from (3.8) the differential cross-section used in Ref. 13 to study oscillation effects in $\bar{\nu}_e$- e scattering.

As a final contribution in this paper, I give an estimate for the helicity transformation effect as neutrinos pass through matter. I start by considering the passage of a Dirac electron type neutrino through matter in which neutrinos and antineutrinos can be produced by reactions. In this matter neutrinos and antineutrinos have the local production densities $\rho_a(x)$ and $\rho_b(x)$ respectively. This is the type of matter one would expect to find within certain stars. It is also assumed that a helicity reversed neutrino interacts with matter as an antineutrino. The corresponding property is assumed also for antineutrinos. The modifications in the formulation without this assumption can be easily made. I use $\alpha(x)$ to represent the absorption coefficient for the process $\nu \longrightarrow \bar{\nu}$ and $\beta(x)$ to represent the absorption coefficient for the process $\bar{\nu} \longrightarrow \nu$. The densities N_a and N_b of neutrinos and antineutrinos respectively found at a distance x from the origin is found from the differential equations

$$\dot{N}_a = -\alpha(x) N_a + \rho_a(x) + \beta(x) N_b$$
$$\dot{N}_b = -\beta(x) N_b + \rho_b(x) + \alpha(x) N_a . \tag{3.10}$$

These equations have the solution

$$N_a = e^{-\int(\alpha+\beta)dx}\left[\int e^{\int(\alpha+\beta)dx}(\beta n(x) + \rho_a)\,dx + C_1\right] \tag{3.11}$$

where

$$n(x) = N_a + N_b = \int(\rho_a + \rho_b)\,dx + C_0 .$$

For the interesting case of terrestrial experiments, an estimate of the helicity transformation effect can be made for the observation of neutrinos in the forward direction at a distance L from the neutrino source. This situation might be realized if neutrinos are observed after passing through a portion of the earth. The cross-section for this case is found from (2.5), (2.7) and

$$\sigma(\lambda_a, \lambda_c, \lambda, m, \Theta=0) = \int \frac{d\sigma}{dt}\frac{dt}{d\Omega}\,\delta(\cos\Theta - 1)\,d\Omega \tag{3.12a}$$

to become

$$\sigma(\lambda_a, \lambda_c, \lambda, m, \Theta=0) = \frac{\omega_a^2 - m^2}{2\pi}\frac{d\sigma}{dt}(\lambda_a, \lambda_c, m, \Theta=0). \tag{3.12b}$$

For both $\lambda = 1$ and $\lambda = -1$, one finds in this case

$$\sigma(\lambda_a, \lambda_a, m, \Theta=0) = \frac{G^2\omega_a^2}{2\pi^2}(|V|^2 + |A|^2)(2 - m^2/\omega_a^2) \tag{3.13a}$$

$$\sigma(\lambda_a, -\lambda_a, m, \Theta=0) = \frac{G^2}{2\pi^2} m^2 |A|^2. \tag{3.13b}$$

For matter of uniform density n_e, one finds for the absorption coefficients

$$\alpha(x) = \beta(x) = n_e\,\sigma(\lambda_a, -\lambda_a, m, \Theta=0).$$

With the initial contitions that $N_a(0)$ is a constant and that $N_b(0)$ is zero, one finds from (3.11) for a distance L from the origin the ratios

$$2\,N_a(L)/N_a(0) = (1 + e^{-Q}) \tag{3.14a}$$

$$2\,N_b(L)/N_a(0) = (1 - e^{-Q}) \tag{3.14b}$$

with

$$Q = 2\sigma(\lambda_a, -\lambda_a, m, \Theta=0)\,\rho_e N_0 L$$

where $N_0 \sim 6.022 \times 10^{23}$ and $n_e = \rho_e N_0$.

Numerical results for the ratios (3.14) and for the parameter $\rho_e m^2 L = 4.739 \times 10^{21}\,Q$ can be determined from Table 1. From this table, one can determine the ratios (3.14) at a distance L(cm) for a given value of Q. The parameter m is the ratio of the neutrino mass to the electron mass. The

The electron density parameter ρ_e has the following approximate values:

$\rho_e \sim 1 - 2$	Sun, Earth
$10^{10} < \rho_e < 10^{13}$	Neutron star
$10^{16} < \rho_e$	Black hole.

Although one can conclude from this numerical estimate that the helicity transformation effect is unlikely to be observed for neutrinos passing through the earth or the sun, the effect may be present in very dense stars if $m \sim 1 - 10^{-1}$. Values in this range are within the experimental bounds for ν_μ or ν_τ, but they are larger than the cosmological bounds[14], $m_e + m_\mu + m_\tau + m_x \sim 40$ eV. If one now considers a condensing star where electron neutrinos are produced from the reaction $n + e \rightarrow p + \nu$ and observes from (3.6) that the probabilities with small oscillation lengths in matter for $\nu_e \leftrightarrow \nu_\mu$ or $\nu_e \longleftrightarrow \nu_\tau$ oscillations are approximately 1/3 - 1/2, then the helicity transformation effect could be signifcant in producing helicity transformed neutrinos in the universe. This effect would have to be considered along with the expected precession of the neutrino's

polarization vector which can occur if the neutrino has a magnetic moment and passes through a dense magnetic field[15]. As a final remark, it is worthwhile to note, until such time that the cosmological bounds on the neutrino masses are better established, that it may be of interest to use the cross-sections (2.5) in looking for (m/ω_a) dependence in precision ν - e scattering.

This research is dedicated to the memory of my friend and colleague Seán Browne. Beannacht Dé leis a anam.

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TABLE 1.

Q	$= 1$	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}
$2N_b(L)/N_a(0)$	$= .632$	$.0952$	$.0099$	10^{-3}	10^{-4}	10^{-5}	10^{-6}

TABLE 1. Neutrino flux ratio at a distance L cm from the origin, $N_a(L) + N_b(L) = N_a(0)$.