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On the Higher Spin Content of Superfields

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Abstract

Two different possibilities of constructing superfields of arbitrary spin content are considered. In one the superfield itself transforms according tom some representation of the homogeneous Lorentz group (H.L.G.) while in the other it is a scalar but is a function of several four component Majorana spinors.

1. Introduction

An important aspect of a superfield [1,2,3] is that it is a supermultiplet field, describing particles of several different spins. Our aim in this paper is to devise means of constructing superfields that contain particles of arbitrarily high spins.

The superfields defined by Wess and Zumino [1] and by Salam and Strathdee [2,3] are themselves Lorentz scalar fields defined over spacetime and the Majorana spinors θ which belong to the $\left[(\frac{1}{2},0)\oplus(0,\frac{1}{2})\right]$ representation of the homogeneous Lorentz group (H.L.G.). Denoting these by

$$\theta_\alpha = [\xi_a \oplus \eta^{\dot{b}}] \; ; \quad \alpha = a, \dot{b} \; ; \quad a, \dot{b} = \pm\tfrac{1}{2} \tag{1}$$

the Majorana restriction is expressed by

$$\xi = \varepsilon\bar{\eta} \quad , \quad \eta = \varepsilon\bar{\xi} \tag{2}$$

where ε is the two-component antisymmetric symbol, or the metric spinor, and $\bar{\eta}$ simply denotes the complex conjugate of η. In addition these Majorana spinors are taken to be anticommuting

$$\{\theta_\alpha , \theta_\beta\} = 0 \tag{3}$$

so that no greater number than four factors of θ can be non-vanishing (or two factors of either ξ or η).

The scalar superfield thus defined can be expanded in powers of θ

$$\begin{aligned}\Phi(x,\theta) = A(x) + \bar{\theta}\psi(x) + \tfrac{1}{4}(\bar{\theta}\theta)\,F(x) \quad &+ (\bar{\theta}\theta)\,\theta\,\chi(x) + \tfrac{1}{32}(\bar{\theta}\theta)^2 D(x) \\ &+ \tfrac{1}{4}(\bar{\theta}\gamma_5\theta)\,G(x) \\ &+ \tfrac{1}{4}(\bar{\theta}i\gamma_\mu\gamma_5\theta)\,A_\mu(x)\end{aligned} \tag{4}$$

in the notation of ref.[3], where A, F and D are scalar fields, G is a pseudo-scalar field, A_μ an axial vector field and ψ and χ are spin-$\frac{1}{2}$ Dirac fields.

Throughout the following, we shall enforce conditions (2) and (3) so that the spacetime translation induced by the supersymmetry [2,3] transformation should be <u>real</u> [2,3].

Below, we introduce two types of superfields which can contain fields of arbitrarily high spins.

The first of these is the non-scalar, covariant superfield

$$\Phi^{(A_1,B_1)\otimes(A_2,B_2)\otimes\cdots}(x,\theta) \tag{5}$$

transforming according to the (in general reducible) representation $(A_1,B_1)\otimes(A_2,B_2)\otimes\cdots$ of the H.L.G. This type of superfield was suggested by Salam and Strathdee [3].

The second type of superfield is a Lorentz scalar, defined over spacetime and <u>several</u> anticommuting four-component Majorana spinors $\theta^{(1)}, \theta^{(2)}, \ldots, \theta^{(n)}$

$$\Phi(x,\theta^{(1)},\theta^{(2)},\ldots,\theta^{(n)}) \,. \tag{6}$$

As seen from the single θ scalar superfield in (4), the spin content of the component fields arises from the magnitude of the "spin" representations of the H.L.G carried byt the Majorana bases. Thus in (4) the highest spin field is the axial vector A corresponding to the basis $\bar{\theta}i\gamma_\mu\gamma_5\theta$. It follows then that for a superfield of the type (5), transforming according to an irreducible representation (I.R.), (A,B), of the H.L.G , the highest spin content will be that of (A,B) itself [4] plus one, i.e. (A+B+1), while the lowest spin will be spin-0 or spin-($|A-B|-1$), whichever is the greater.

This means that with a judicious choice of (A,B), the superfield of spins up to (A+B+1) will be a tower of particles starting from spin-0. On the other hand, if A-B is not sufficiently small, for example for a Joos-Weinberg [5,6] (J-W) representation where A=j, B=0 with $j > 1$, the peculiar situation will occur where the lowest spin will not be zero and the supermultiplet will occupy a spin band. This situation however need not arise, and by simply constructing superfields of reducible representations (5) the spin towers can be made to start from spin-0.

Another way of ensuring that the tower of spins starts from zero is to construct superfields of the type indicated by (5). This last type has the advantage that the superfield itself is a Lorentz scalar, but the disadvantage that the Majorana bases are much more copious owing to the loss of the restrictions arising from the use of identical Majorana spinors.

In the two following sections both approaches are presented respectively.

2. Covariant Superfields

2.1 The Covariance

According to the form (5), a covariant superfield labelled by the I.R. (A,B) of H·L·G transforms, under weak transformation Λ and supersymmetry "rotation" ε, according to

$$U(\Lambda)\,\Phi^{(A,B)}(x,\theta)\,U^{-1}(\Lambda) = D^{(A,0)}(\Lambda^{-1}) \otimes D^{(0,B)}(\Lambda^{-1})\,\Phi^{(A,B)}(\Lambda x,\, D^{(\frac{1}{2})}\theta) \tag{7}$$

$$U(\varepsilon)\,\Phi^{(A,B)}(x,\theta)\,U^{-1}(\varepsilon) = \Phi^{(A,B)}(x + \tfrac{i}{2}\,\bar{\varepsilon}\gamma_\mu\theta,\ \theta+\varepsilon) \tag{8}$$

where we have used the notation of ref. [6]

$$D^{(\frac{1}{2})}(\Lambda) = \begin{bmatrix} D^{(\frac{1}{2},0)}(\Lambda) & 0 \\ 0 & D^{(0,\frac{1}{2})}(\Lambda) \end{bmatrix}, \quad \gamma_\mu = \begin{bmatrix} 0 & -i\sigma_\mu \\ -i\tilde{\sigma}_\mu & 0 \end{bmatrix}, \quad \sigma_\mu = 1, \vec{\sigma}\,,\ \tilde{\sigma}_\mu = 1, -\vec{\sigma}\,. \tag{9}$$

It is a straightforward matter to decompose the superfield specified by (7) and (8), in the manner of (4) by using all the available independent Majorana bases, whose coefficients then will be the fields of different spins. These bases are presented in the next subsection.

2.2 The Majorana Bases

Here we construct the definite parity [2,3] bases from the spinor θ, subject to the conditions (2) and (3). The following basic spinor identities are used in the reduction of these bases.

$$\varepsilon\,\sigma_\mu\,\varepsilon = -\tilde{\sigma}^T \tag{10}$$

$$\varepsilon_{aa'}\,\varepsilon_{bb'} = \varepsilon_{ab}\,\varepsilon_{a'b'} + \varepsilon_{ab'}\,\varepsilon_{ba'} \tag{11}$$

$$\sigma_\mu \tilde{\sigma}_\nu = g_{\mu\nu} + \tfrac{1}{2} i\, \varepsilon_{\mu\nu\rho\sigma}\, \sigma^\rho \tilde{\sigma}^\sigma \tag{12a}$$

$$\tilde{\sigma}_\mu \sigma_\nu = g_{\mu\nu} - \tfrac{1}{2} i\, \varepsilon_{\mu\nu\rho\sigma}\, \sigma^\rho \tilde{\sigma}^\sigma . \tag{12b}$$

For example, using (2), (3) and (10) one finds that

$$\bar{\xi}_1 \tilde{\sigma}_\mu \xi_2 = - \bar{\eta}_2 \sigma_\mu \eta_1 \;,$$

and using the notation (6)

$$\bar{\Psi} = \Psi^\dagger \beta \,, \quad \beta = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} , \quad \gamma_5 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{13}$$

this means that $\bar{\theta} i \gamma_\mu \theta$ vanishes identically while $\bar{\theta} i \gamma_\mu \gamma_5 \theta = 2\bar{\eta}\sigma_\mu \eta$ is non-vanishing.

We list now the bases constructed from θ, both in four-component θ spinors and in two-componenet spinors ξ and η:

$$B^{(0)} = 1 \tag{14a}$$

$$B^{(1)}_1 = \bar{\theta} = [\bar{\eta}, \bar{\xi}] \,, \; B^{(1)}_2 = \bar{\theta} i \gamma_\mu = [\bar{\xi}\tilde{\sigma}_\mu , \bar{\eta}\sigma_\mu] \,, \; B^{(1)}_3 = \bar{\theta}\gamma_{[\mu}\gamma_{\nu]} = [\bar{\eta}\, \sigma_{[\mu}\tilde{\sigma}_{\nu]} , \bar{\xi}\, \tilde{\sigma}_{[\mu}\sigma_{\nu]}] \tag{14b}$$
$$\bar{B}^{(1)}_i = B^{(1)}_i \gamma_5 \quad , \; i = 1,2,3$$

$$B^{(2)}_1 = \bar{\theta}\theta = (\bar{\eta}\cdot\xi + \bar{\xi}\cdot\eta) \,, \; B^{(2)}_2 = \bar{\theta}\gamma_5\theta = (\bar{\eta}\cdot\xi - \bar{\xi}\cdot\eta) \,, \; B^{(2)}_3 = \bar{\theta} i \gamma_\mu \gamma_5 \theta = 2\bar{\eta}\sigma_\mu \eta \tag{14c}$$

$$B^{(3)}_{ij} = B^{(2)}_i \otimes B^{(1)}_j \qquad i,j = 1,2,3 \tag{14d}$$

$$B^{(4)}_{ij} = B^{(2)}_i \otimes B^{(2)}_j \qquad i,j = 1,2,3 \,, \tag{14e}$$

where not all $B^{(3)}_{ij}$ and $B^{(4)}_{ij}$ are independent or non-vanishing, and the forms for each of these are listed in the tables below, computed by means of identities (10), (11), (12) and restrictions (2) and (3):

$$B^{(3)}_{ij} \tag{14d}$$

$(\bar\theta\theta)\bar\theta=[(\bar\xi\cdot\eta)\bar\eta\,,(\bar\eta\cdot\xi)\bar\xi]$	$(\bar\theta\theta)\bar\theta i\gamma_\mu=[(\bar\eta\cdot\xi)\bar\xi\tilde\sigma_\mu\,,(\bar\xi\cdot\eta)\bar\eta\sigma_\mu]$	$(\bar\theta\theta)\bar\theta\gamma_{[\mu}\gamma_{\nu]}=[(\bar\eta\cdot\xi)\bar\xi\tilde\sigma_{[\mu}\sigma_{\nu]}\,,(\bar\xi\cdot\eta)\bar\eta\sigma_{[\mu}\sigma_{\nu]}]$
$(\bar\theta\gamma_5\theta)\bar\theta=(\bar\theta\theta)\bar\theta\gamma_5$ $=[(\bar\xi\cdot\eta)\bar\eta\,,-(\bar\eta\cdot\xi)\bar\xi]$	$(\bar\theta\gamma_5\theta)\bar\theta i\gamma_\mu=(\bar\theta\theta)\bar\theta\gamma_5 i\gamma_\mu$ $=[(\bar\eta\cdot\xi)\bar\xi\tilde\sigma_\mu\,,-(\bar\xi\cdot\eta)\bar\eta\sigma_\mu]$	$(\bar\theta\gamma_5\theta)\bar\theta\gamma_{[\mu}\gamma_{\nu]}=(\bar\theta\theta)\bar\theta\gamma_5\gamma_{[\mu}\gamma_{\nu]}$ $=[(\bar\eta\cdot\xi)\bar\xi\tilde\sigma_{[\mu}\sigma_{\nu]}\,,-(\bar\xi\cdot\eta)\bar\eta\sigma_{[\mu}\tilde\sigma_{\nu]}]$
$(\bar\theta i\gamma_\mu\gamma_5\theta)\bar\theta=-(\bar\theta\theta)\bar\theta\gamma_5 i\gamma_\mu$ $=[(\bar\eta\cdot\xi)\bar\xi\tilde\sigma_\mu\,,-(\bar\xi\cdot\eta)\bar\eta\sigma_\mu]$	$(\bar\theta\gamma_\mu\gamma_5\theta)\bar\theta\gamma_\nu=-(\bar\theta\theta)\bar\theta\gamma_5\gamma_\mu\gamma_\nu$ $=[-(\bar\xi\cdot\eta)\bar\eta\sigma_{[\mu}\tilde\sigma_{\nu]}\,,(\bar\eta\cdot\xi)\bar\xi\tilde\sigma_{[\mu}\sigma_{\nu]}]$	$(\bar\theta i\gamma_\mu\gamma_5\theta)\bar\theta\gamma_{[\rho}\gamma_{\nu]}=(\bar\theta\theta)\bar\theta\gamma_5 i\gamma_\mu\gamma_{[\rho}\gamma_{\nu]}$ $=[(\bar\eta\cdot\xi)\bar\xi\tilde\sigma_\mu\sigma_{[\rho}\tilde\sigma_{\nu]}\,,-(\bar\xi\cdot\eta)\bar\eta\sigma_\mu\tilde\sigma_{[\rho}\sigma_{\nu]}]$

$$B^{(4)}_{ij} \tag{14e}$$

$(\bar\theta\theta)^2=2(\bar\eta\cdot\xi)(\bar\xi\cdot\eta)$	$(\bar\theta\theta)(\bar\theta\gamma_5\theta)=0$	$(\bar\theta\theta)(\bar\theta i\gamma_\mu\gamma_5\theta)=0$
$(\bar\theta\gamma_5\theta)(\bar\theta\theta)=0$	$(\bar\theta\gamma_5\theta)^2=-2(\bar\eta\cdot\xi)(\bar\xi\cdot\eta)$	$(\bar\theta\gamma_5\theta)(\bar\theta i\gamma_\mu\gamma_5\theta)=0$
$(\bar\theta i\gamma_\mu\gamma_5\theta)(\bar\theta\theta)=0$	$(\bar\theta i\gamma_\mu\gamma_5\theta)(\bar\theta\gamma_5\theta)=0$	$(\bar\theta\gamma_\mu\gamma_5\theta)(\bar\theta\gamma_\nu\gamma_5\theta)=(\bar\theta\theta)^2 g_{\mu\nu}$ $=2(\bar\eta\cdot\xi)(\bar\xi\cdot\eta)g_{\mu\nu}$

It is clear that many of these bases will not be effectively independent (even if they are non-vanishing) for some of these are simply functions of the invariant tensors $g_{\mu\nu}$ and $\varepsilon_{\mu\nu\rho\sigma}$. An identity derived from (12) is useful to make these forms explicit:

$$\sigma_\mu\tilde\sigma_\nu\sigma_\rho = i\,\varepsilon_{\mu\nu\rho\sigma}\sigma^\sigma + g_{\mu\nu}\sigma_\rho - g_{\mu\rho}\sigma_\nu + g_{\nu\rho}\sigma_\mu \tag{15a}$$

$$\tilde\sigma_\mu\sigma_\nu\tilde\sigma_\rho = -i\,\varepsilon_{\mu\nu\rho\sigma}\tilde\sigma^\sigma + g_{\mu\nu}\tilde\sigma_\rho - g_{\mu\rho}\tilde\sigma_\nu + g_{\nu\rho}\tilde\sigma_\mu \,. \tag{15b}$$

2.3 Two Examples

To illustrate the construction and decomposition of the covariant superfields given in 2.1 by means of the bases in 2.2, we consider two simple examples, one a Lorentz-vector superfield with $A=B=\frac{1}{2}$, and the other a Dirac-spinor superfield that transforms according to the $(\frac{1}{2},0)+(0,\frac{1}{2})$ representation of the H·L·G.

<u>Vector Superfield</u>: Using the bases (14) we decompose $V_\mu(x,\theta)$ as follows:

$$\begin{aligned} V_\mu(x,\theta) = V_\mu^{(1)} + \bar\theta^\alpha \psi^{(1)}_{\mu\alpha} \quad & +(\bar\theta\theta)V_\mu^{(2)} \quad && +(\bar\theta\theta)\,\bar\theta^\alpha\psi^{(2)}_{\mu\alpha} \quad +(\bar\theta\theta)^2 V_\mu^{(3)}. \\ +(\bar\theta i\gamma_\mu)^\alpha \psi^{(1)}_\alpha \quad & +(\bar\theta\gamma_5\theta)A_\mu && +(\bar\theta\theta)(\bar\theta i\gamma_\mu)^\alpha\psi^{(2)}_\alpha \\ & +(\bar\theta i\gamma_\mu\gamma_5\theta)P \\ & +(\bar\theta i\gamma_\nu\gamma_5\theta)H_{\mu\nu} \end{aligned} \qquad (16)$$

Here P is a pseudoscalar field, A_μ an axial vector field containing a (spin)<u>parity</u> 1^+ and a 0^-particle, $V_\mu^{(i)}$ $(i=1,2,3)$ are vector fields each containing a 1^- and a 0^+ particle, $_{\mu\nu}$ is a tensor field whose symmetric part includes a 2^- and a 0^- particle and its antisymmetric part a 1^+ particle. $\psi^{(1)}$ and $\psi^{(2)}$ are spin-$\frac{1}{2}$ Dirac fields while $\psi_\mu^{(1)}$ and $\psi_\mu^{(2)}$ are <u>unrestricted</u> Rarita-Schwinger[7] (R-S) fields, each containing a spin-3/2 and a spin-$\frac{1}{2}$ particle.

As explained in Section 1, the maximum spin here is spin-(A+B+1)=2, while the minimum is spin-0.

In carrying out the decomposition (16) we have used the fact that the invariant tensors and the γ-matrices(*) in the Majorana bases serve to project out the higher spin content of certain fields, for example the following terms containing the R-S field ψ^μ and a third-rank tensor field $T^{\mu\nu\lambda}$,

$$\bar\theta\gamma_\nu\gamma_\mu\psi^\mu = \bar\theta\gamma_\nu\psi \quad \text{and} \quad (\bar\theta\theta)^2 g_{\mu\nu}T^{\mu\nu\lambda} = (\bar\theta\theta)^2 V^\lambda$$

effectively contain the Dirac ψ and vector V^λ fields respectively, and are then simply absorbed into the corresponding field with the same Majorana basis.

(*) which play the role of Clebsch-Gordan coefficients here.

Just for ordinary local fields, if the vector superfield $V_\mu(x,\theta)$ is subjected to the Lorentz condition

$$\partial_\mu V_\mu(x,\theta) = 0 \tag{17}$$

its spin content diminishes appreciatly. In fact, imposing (17) on (16) gives rise to the restrictions

$$\begin{aligned}
&\partial_\mu \psi^{(1)}_\mu + \psi^{(1)} = 0\\
&\partial_\mu \psi^{(2)}_\mu + \psi^{(2)} = 0\\
&\partial_\mu V^{(i)}_\mu = 0 \quad i=1,2,3\\
&\partial_\mu A_\mu = 0\\
&\partial_\mu P + \partial_\nu H_{\nu\mu} = 0 .
\end{aligned} \tag{18}$$

It follows from the restrictions (18) that the number of spin- $\frac{1}{2}$ particles is reduced by two, the number of scalars by three, the pseudoscalars by one, and the number of vectors by one.

<u>Dirac Superfield</u>: Using the basis (14) we decompose the $\left[(\frac{1}{2},0) \oplus (0,\frac{1}{2})\right]$ superfield

$$\Psi_\alpha(x,\theta) = [\, \Phi_a(x,\theta) \oplus \chi^{\dot b}(x,\theta)] \quad ; \ \alpha = a,\dot b \ ; \quad a,\dot b = \pm\tfrac{1}{2}$$

in the following way:

$$\begin{aligned}
\Psi_\alpha(x,\theta) = \psi^{(1)}_\alpha + \theta_\alpha S^{(1)} \quad &+ (\bar\theta\theta)\,\psi^{(2)}_\alpha \quad &&+ (\bar\theta\theta)\,\theta_\alpha S^{(2)} \quad + (\bar\theta\theta)^2 \psi^{(4)}_\alpha \cdot\\
+ (\gamma_5\theta)_\alpha P^{(1)} \quad &+ (\bar\theta\gamma_5\theta)\,\psi^{(3)}_\alpha &&+ (\bar\theta\gamma_5\theta)\,\theta_\alpha P^{(2)}\\
+ (i\gamma_\mu\theta)_\alpha V^{(1)}_\mu \quad &+ (\bar\theta i\gamma_\mu\gamma_5\theta)\,\psi_{\mu\alpha} &&+ (\bar\theta\theta)(i\gamma_\mu\theta)_\alpha V^{(2)}_\mu\\
+ (i\gamma_\mu\gamma_5\theta)_\alpha A^{(1)}_\mu \quad & &&+ (\bar\theta\gamma_5\theta)(i\gamma_\mu\theta)\, A^{(2)}_\mu\\
+ (\gamma_{[\mu}\gamma_{\nu]}\theta)_\alpha F^{(1)}_{\mu\nu} \quad & &&+ (\bar\theta\theta)(\gamma_{[\mu}\gamma_{\nu]}\theta)_\alpha F^{(2)}_{\mu\nu}\\
+ (\gamma_{[\mu}\gamma_{\nu]}\gamma_5\theta)_\alpha G^{(1)}_{\mu\nu} \quad & &&+ (\bar\theta\gamma_5\theta)(\gamma_{[\mu}\gamma_{\nu]}\theta)_\alpha G^{(2)}_{\mu\nu}
\end{aligned} \tag{19}$$

As expected the highest spin occuring in this supermultiplet is the spin-3/2 content of the R-S field ψ^{μ}_{α}, while the lowest is spin-0. The nomenclature used in (19) is the same as that used in (18). Thus the boson content is, two scalars, two pseudoscalars, two vectors, two axial vectors, and four pure-spin-1 fields (two of either parity signature). The fermion content is more subtle and needs comment. Apart from the four spin-$\frac{1}{2}$ Dirac fields exhibited, the unrestricted R-S field contains a spin-$\frac{1}{2}$ Dirac field in addition.

Now it might appear that in the exapansion (19), the terms

$$(\bar{\theta}\gamma_5\psi)\,\theta \quad \text{and} \quad (\bar{\theta}\psi)\,\gamma_5\theta \tag{20}$$

are omitted*). It turns out that these terms are not independent of each other, and from (11) it follows that

$$(\bar{\theta}\psi)\,\gamma_5\theta + (\bar{\theta}\gamma_5\psi)\,\theta = -\tfrac{1}{4}(\bar{\theta}\gamma_5\theta)\,\psi - \tfrac{1}{4}(\bar{\theta}\theta)\,\gamma_5\psi \tag{21}$$

which would simply be obsorbed into the appropriate terms in (19), while **)

$$(\bar{\theta}\psi)\,\gamma_5\theta - (\bar{\theta}\gamma_5\psi)\,\theta = 2(\bar{\theta}\, i\gamma_\mu\gamma_5\theta)\,\psi_\mu(\tfrac{1}{2}) \tag{22}$$

where $\psi_\mu(\frac{1}{2})$ is the spin-$\frac{1}{2}$ part of the unrestricted R-S field given by

$$\psi^{\mu}_{\alpha}(\tfrac{1}{2}) = (\gamma^{\mu}\psi)_\alpha\ , \tag{23}$$

which is already accounted for by ψ^{μ} in (19).

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*) Other terms like $(\bar{\theta}\theta)(\bar{\theta}\cdot\psi)\theta = -\frac{1}{4}(\bar{\theta}\theta)^2\psi$ are already accounted for.

**) Using the wellknown spinor identities $\sigma^{\mu}_{a\dot{b}}\,\tilde{\sigma}_{\mu}^{\dot{d}c} = 2\,\delta_{\dot{b}}^{\dot{d}}\,\delta_a^c$, $\sigma^{\mu}_{a\dot{b}}(\tilde{\sigma}_\mu)_{c\dot{d}} = 2\,\varepsilon_{ac}\,\varepsilon_{\dot{b}\dot{d}}$

The Dirac superfield (19) has the same H·L·G transformation properties as the ordinary Dirac field, as well as the property that under parity its two-component spinor parts will undergo the transformations

$$\Phi_a(x,\theta) \rightleftarrows X^b(x,\theta) . \tag{24}$$

Just as in the case of the free Dirac field, Φ and X of the free superfield are related through the Dirac equation

$$(\gamma\cdot\partial - m)\Psi(x,\theta) = 0 . \tag{25}$$

Finally therefore, imposition of the conditions (25) gives rise to the following restrictions on the fields contained in $\Psi(x,\theta)$,

$$\begin{array}{ll} S^{(i)} = i\,\partial_\mu V_\mu^{(i)} & P^{(i)} = i\,\partial_\mu A_\mu^{(i)} \\ i\,V_\mu^{(i)} = \partial_\mu S^{(i)} + 4\,\partial_\nu F_{\mu\nu}^{(i)} & i\,A_\mu^{(i)} = \partial_\mu P^{(i)} + 4\,\partial_\nu G_{\mu\nu}^{(i)} \\ F_{\mu\nu}^{(i)} = \frac{i}{4}\left(\partial_\mu V_\nu^{(i)} - \partial_\nu V_\mu^{(i)}\right) & G_{\mu\nu}^{(i)} = \frac{i}{4}\left(\partial_\mu A_\nu^{(i)} - \partial_\nu A_\mu^{(i)}\right) \end{array} \tag{26}$$

both for i=1 and i=2 (cf. labels in (19)). This means that out of the three sets of fields, one scalar (pseudoscalar), one vector (axial vector) and one pure-spin-one, only one, say the vector (axial vector) can be chosen as independent, thus diminishing the content of the supermultiplet Ψ .

The above decomposition (19) of the Dirac superfield is the simplest case of a J-W 2(2j+1)-component superfield [8] decomposition, to which there will corespond the more general parity constraints

$$\left(\gamma^{\mu_1\cdots\mu_{2j}}\partial_{\mu_1}\cdots\partial_{\mu_{2j}} - m^{2j}\right)\Psi^{[(j,0)\oplus(0,j)]}(x,\theta) = 0 \tag{27}$$

where the 2(2j+1) x 2(2j+1) spin-matrices $\gamma^{\mu_1\cdots\mu_{2j}}$ as well as their relevant properties are given in detail in ref. (6).

2.4 Choice of Representation

The question of the choice of the H·L·G representation according to which a superfield transforms is an open one just like in the case of space-time-local fields.

As remarked in Section 1, for a covariant-superfield to contain particles of spins starting from spin-zero, it must transform according to an I·R of H·L·G satisfying $|A-B| \leqslant 1$.

Another aspect we comment on is that due to the H·L·G transformation properties of the Majorana bases (14), it is impossible to avoid component-fields of the tensor and (generalised) R-S type, even if the superfield itself might be a J-W type, for example the Dirac superfield considered above. Conversely a tensor or R-S type superfield will involve component fields of the J-W types, for example in the vector superfield above the field $H_{\mu\nu}$ has an antisymmetric part $G_{\mu\nu}$ which is essentially a $[(1,0) \oplus (0,1)]$ J-W representation, namely

$$G_{\mu\nu} = \varphi_\alpha(1,0)\, \sigma^\alpha(0,1)^*_{\mu\nu} + \chi^{\dot\beta}(0,1)\, \sigma_{\dot\beta}(1,0)^*_{\mu\nu}$$

$$\sigma^\alpha(0,1)^*_{\mu\nu} = \frac{1}{\sqrt{2}} \left[\tfrac{1}{2}\tfrac{1}{2}1\right]^\alpha_{aa'} (\varepsilon\, \sigma_\nu \tilde{\sigma}_\mu)^{aa'}, \tag{28}$$

where the symbol $\left[\frac{1}{2}\,\frac{1}{2}\,1\right]$ stands for a Clebsch-Gordan coefficient.

3. Scalar Several-θ Superfields

3.1 The Fields

The following construction yields Lorentz scalar superfields that are supermultiplets of fields of particles with spins ranging from zero to arbitrarily high values.

We extend the definition of a superfield, by considering fields that are functions of space-time and in general of n four-component Majorana spinors $\theta^{(1)}, \theta^{(2)}, \ldots, \theta^{(i)}, \ldots \theta^{(n)}$ which anticommute

$$\{\theta_\alpha^{(i)}, \theta_\beta^{(j)}\} = 0 \tag{29}$$

for any i and j, from 1...n. We actually require that the corresponding generators $S^{(1)}, S^{(2)}, \ldots, S^{(i)}, \ldots, S^{(n)}$ of the supersymmetry transformations satisfy the following anticommutation relations

$$\{S_\alpha^{(i)}, S_\beta^{(j)}\} = 0 \quad , \quad i \neq j$$

$$\{S_\alpha^{(i)}, S_\beta^{(i)}\} = (\gamma_\mu C)_{\alpha\beta} P_\mu \ . \tag{30}$$

We further require that each $S^{(i)}$ satisfy the same commutation relations with the generators P_μ and $J_{\mu\nu}$ of the Poincaré group as given in Refs.(2,3). The supersymmetry transformation

$$\theta^{(i)} \to \theta^{(i)} + \varepsilon^{(i)} \quad , \quad i = 1, \ldots, n \tag{31a}$$

then causes x_μ to be translated according to

$$x_\mu \to x_\mu + \frac{i}{2} \sum_{i=1}^{n} \bar{\varepsilon}^{(i)} \gamma_\mu \theta^{(i)} \ . \tag{31b}$$

Following the procedure in Ref. (3), the variations $\delta\Phi$ due to infinitesimal versions of the transformations (9), and hence also the form of the "covariant derivatives", can be calculated. The "covariant derivative" of each $\theta^{(i)}_{\alpha}$ is clearly then of the same form as that for the single θ superfield.

3.2 The Highest Spin Content

Here we do not give complete expansions in a Majorana basis as we did for covariant-superfields, mainly because the procedure is similar, straightforward and cumbersome. Instead we examine the maximum spin content of the several-θ superfields.

The Majorana bases will consist of a set of the form (14) for each $\theta^{(i)}$, in addition to the bases constructed from several different spinors $\theta^{(i)}, \theta^{(j)}, \ldots$ $(i \neq j \neq \ldots)$. Compared to (14) these new sets of bases will have more independent members as the restrictions ensuring from the use of identical spinors θ will be relaxed. To start with, the vector bases $\theta^{(i)} i\gamma_\mu \theta^{(j)}$ will be nonzero for $i \neq j$.

The maximum power of θ's in the expansion of this superfield will be four for each type of θ, namely $4n$.

Just as in Section 2, each $\theta^{(i)}$ here is taken as being subject to the Majorana conditions (3), which results in <u>halving</u> the number of independent elements in the (otherwise spin-$\frac{1}{2}$) θ's, thus the highest (irreducible) spin basis that can be constructed from n types of θ's is n. In particular for a single θ scalar superfield[1,2,3], the highest spin bases is spin-one. We give a demonstration of this below for a two-θ scalar superfield. For simplicity, we use the two-component formalism.

Denoting the two spinors by

$$\theta = [\xi \oplus \eta] \quad \text{and} \quad \theta' = [\xi' \oplus \eta']$$

we consider the following bases constructed from both θ's:

$$B_1 = (\bar{\eta}\sigma_\mu\eta')(\bar{\eta}\sigma_\nu\eta') \tag{32a}$$

$$B_2 = (\bar{\eta}\sigma_\mu\eta)(\bar{\eta}\sigma_\nu\eta') \tag{32b}$$

$$B_3 = (\bar{\eta}\sigma_\mu\eta')(\bar{\eta}'\sigma_\nu\eta) \; . \tag{32c}$$

It follows from the properties (2), (3) and (29) of θ and θ', and identities (10) and (11) that

$$B_1 = (\bar{\xi}'\cdot\eta')(\bar{\eta}\cdot\xi)\, g_{\mu\nu} \tag{33a}$$

$$B_2 = -(\bar{\eta}\sigma_\nu\eta)(\bar{\eta}\sigma_\mu\eta') \tag{33b}$$

$$B_3 = -(\bar{\eta}\sigma_\mu\eta)(\bar{\eta}'\sigma_\nu\eta') + (\bar{\xi}'\cdot\eta)(\bar{\eta}\sigma_\mu\tilde{\sigma}_\nu\xi') \; . \tag{33c}$$

Clearly (33a) carries the H·L·G transformations of a particle of only spin-zero, (33b) being antisymmetric in μ and ν carries only spin-one, while (33c) can carry the transformation of a particle of spin-2.

Thus using a two-θ scalar superfield we can describe particles of spin-2 while with a one-θ scalar superfield one had only up to spin-1. That no higher spin basis can be constructed with only two θ's is easily checked. For example multiplying the spin-2 basis B_3 by $(\bar{\eta}\sigma_\lambda\eta)$ results (cf. (33b)) in antisymmetry between all three indices μ, ν, λ, and such a tensor cannot carry a totally symmetric third rank spin-3 tensor field. Multiplying B_3 by $(\bar{\eta}\sigma_\lambda\eta')$ on the other hand results (cf. (33a)) in $(\bar{\eta}\sigma_\nu\eta)g_{\mu\lambda}$ which could also not carry a spin-3 field.

We make a final remark. Amongst the bases that can be constructed with n θ's are the J-W bases transforming like the I·R (j,o) of H·L·G, for all j up to $\frac{n}{2}$. For n=2 these are, the (1,0) bases

$$[\tfrac{1}{2}\tfrac{1}{2}1]^{aa'}_{\alpha}\ \xi^{(1)}_{a}\ \xi^{(2)}_{a'}$$

and its parity conjugate, the (0,1) basis

$$[\tfrac{1}{2}\tfrac{1}{2}1]^{\dot{\beta}}_{\dot{b}\dot{b}'}\ \eta^{\dot{b}}_{(1)}\ \eta^{\dot{b}'}_{(2)}\ .$$

With only n types of $\theta^{(i)}$, it is impossible to construct in this manner a non-vanishing $(\frac{1}{2}(n+1),0)$ basis. This follows from (29), the symmetry of $\left[\frac{1}{2}\,\frac{1}{2}\ 1\right]^{aa'}_{\alpha}$ in the interchange of a and a', and the properties of the angular momentum re-coupling coefficients.

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